Notes from the artist

I'm hoping this book will expose younger students to concepts they normally wouldn't see until higher grades. And that it will give advanced students some new views of concepts they're already familiar with.

Thank you to Steven Pietrobon for his many helpful comments. Also to Isaac Kuo for his suggestion. They've helped me make this a better book. Any mistakes in this book are my own.

Hollister (Hop) David

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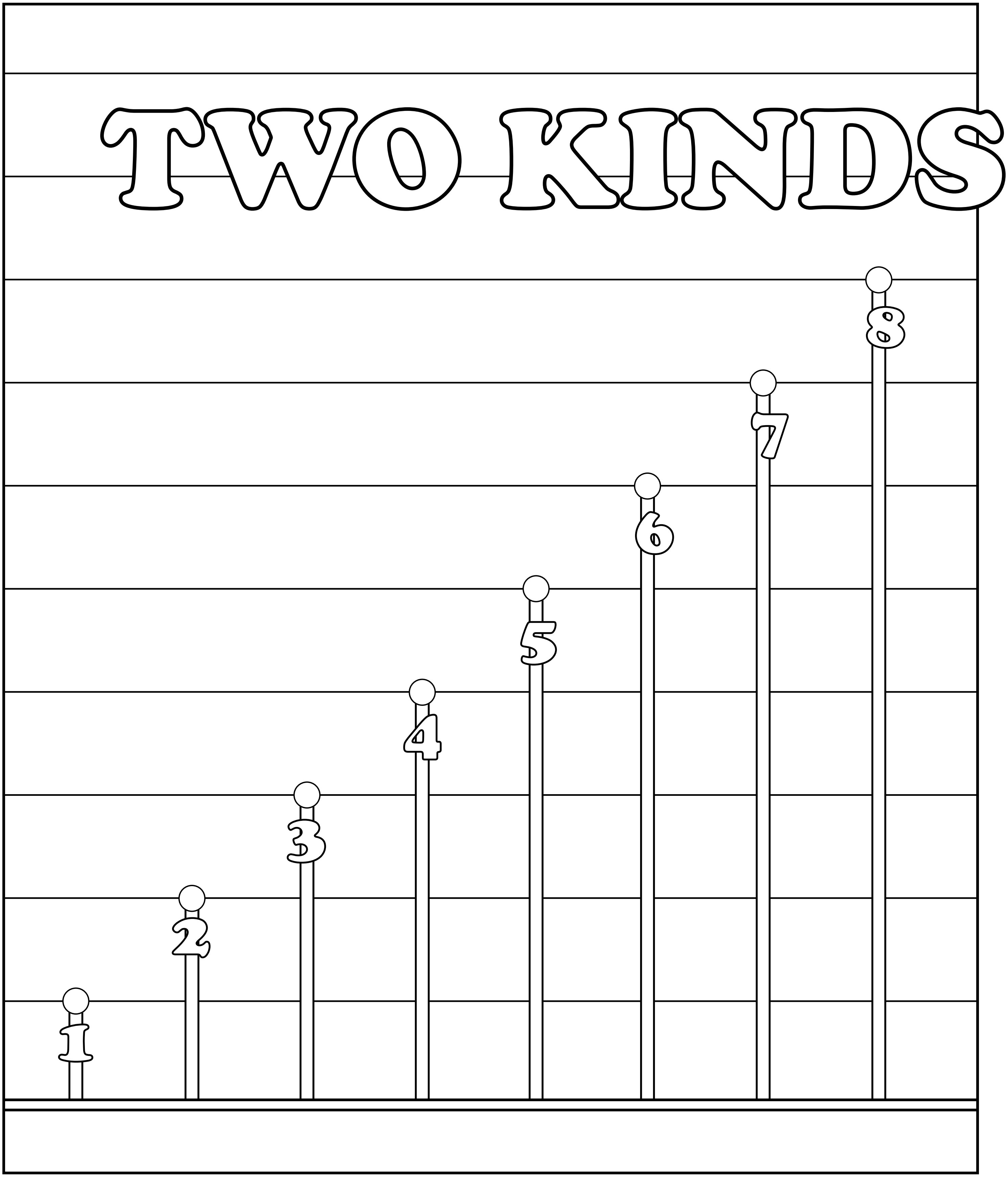
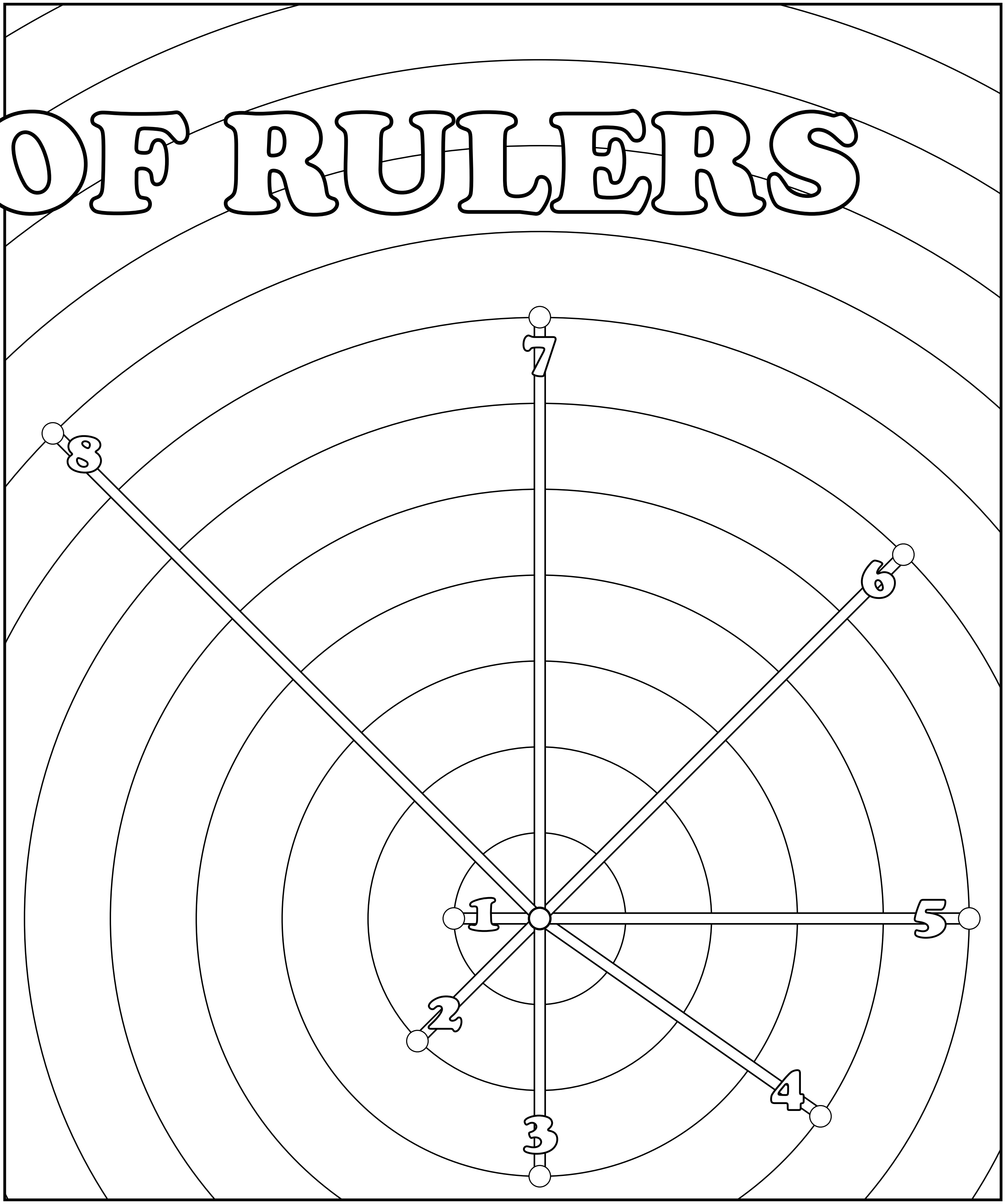
Any corrections, suggestions or comments, please contact me at hopd@cunews.info

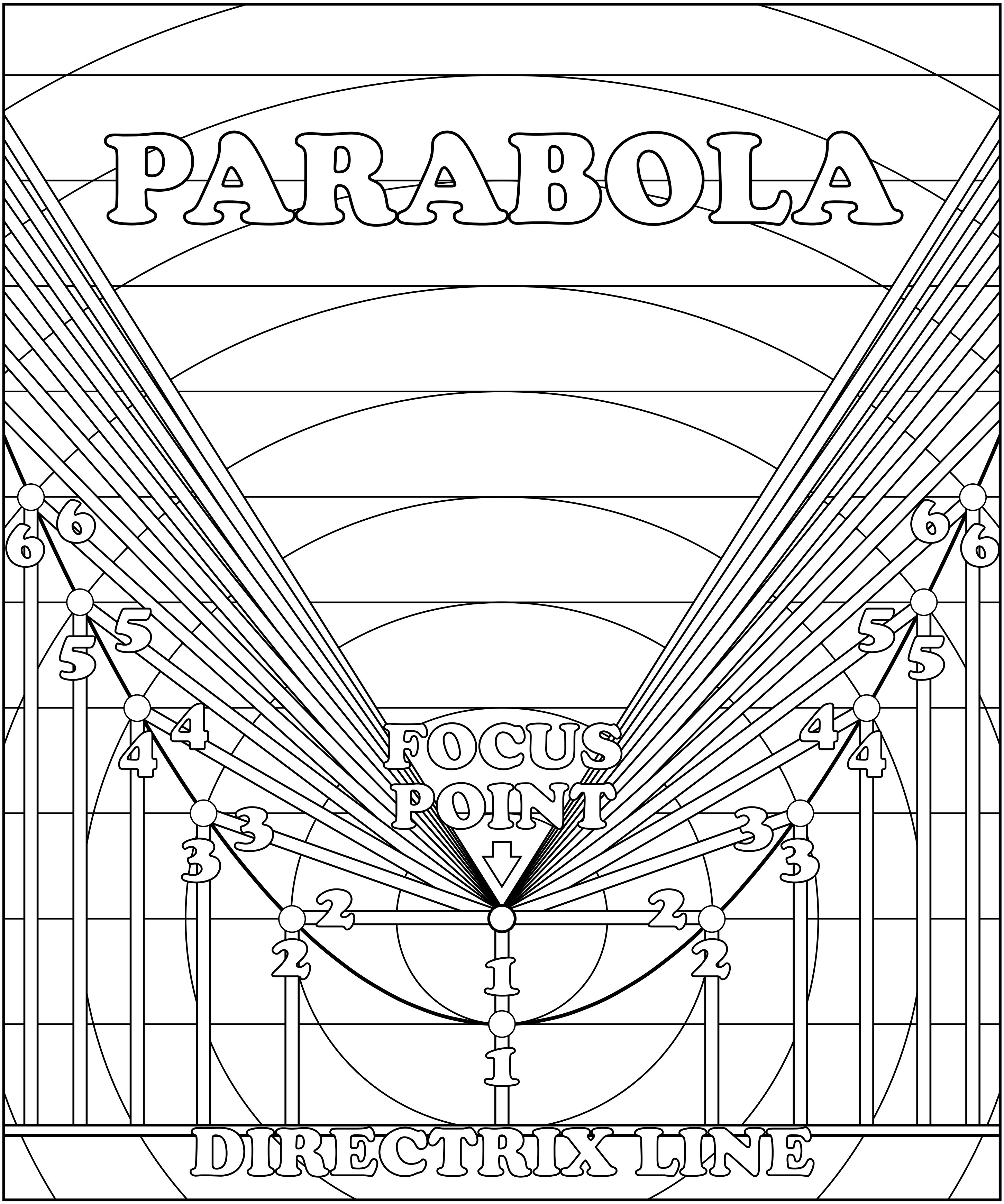


Evenly spaced parallel lines
measure distance from a line.

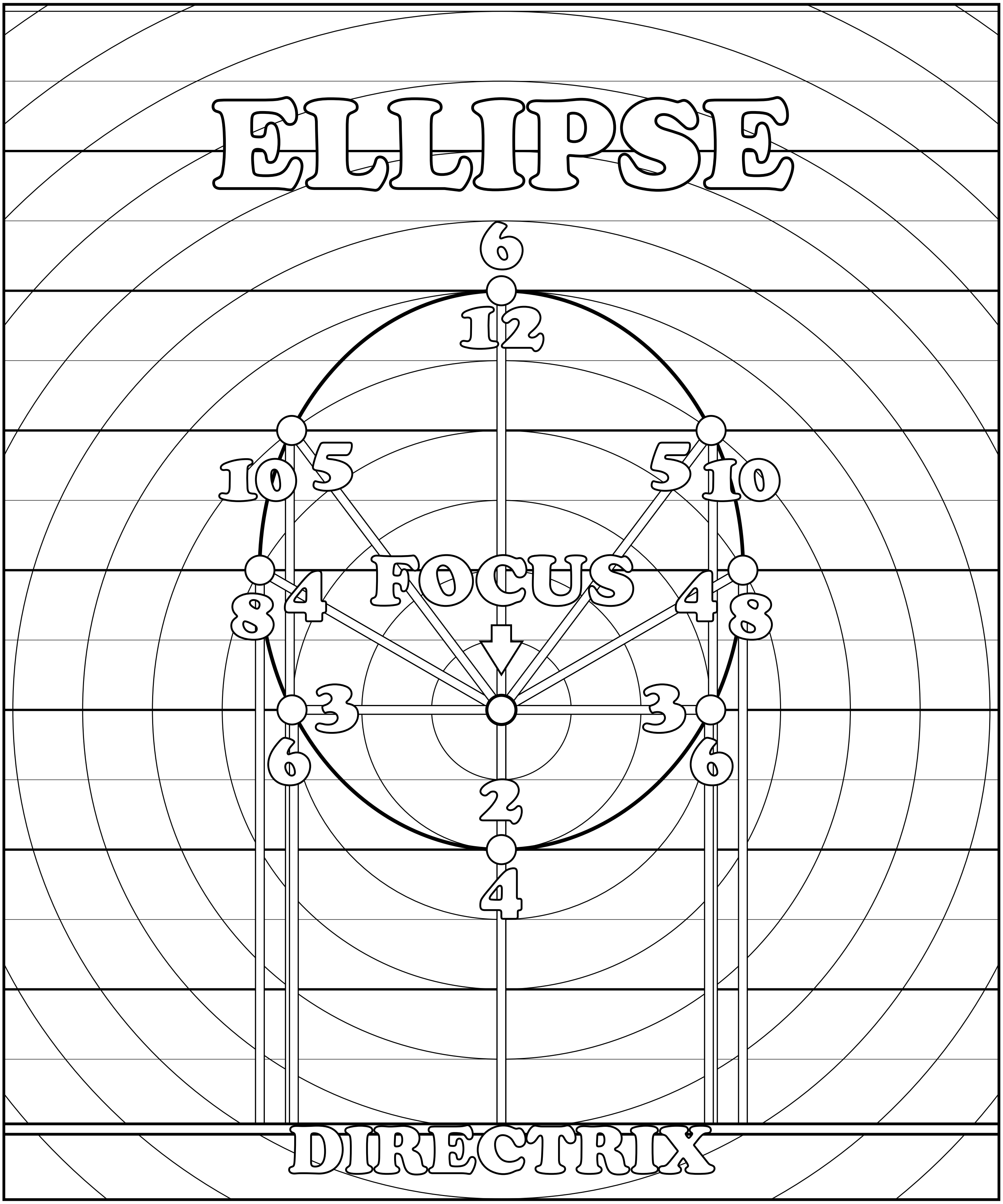
OF RULERS



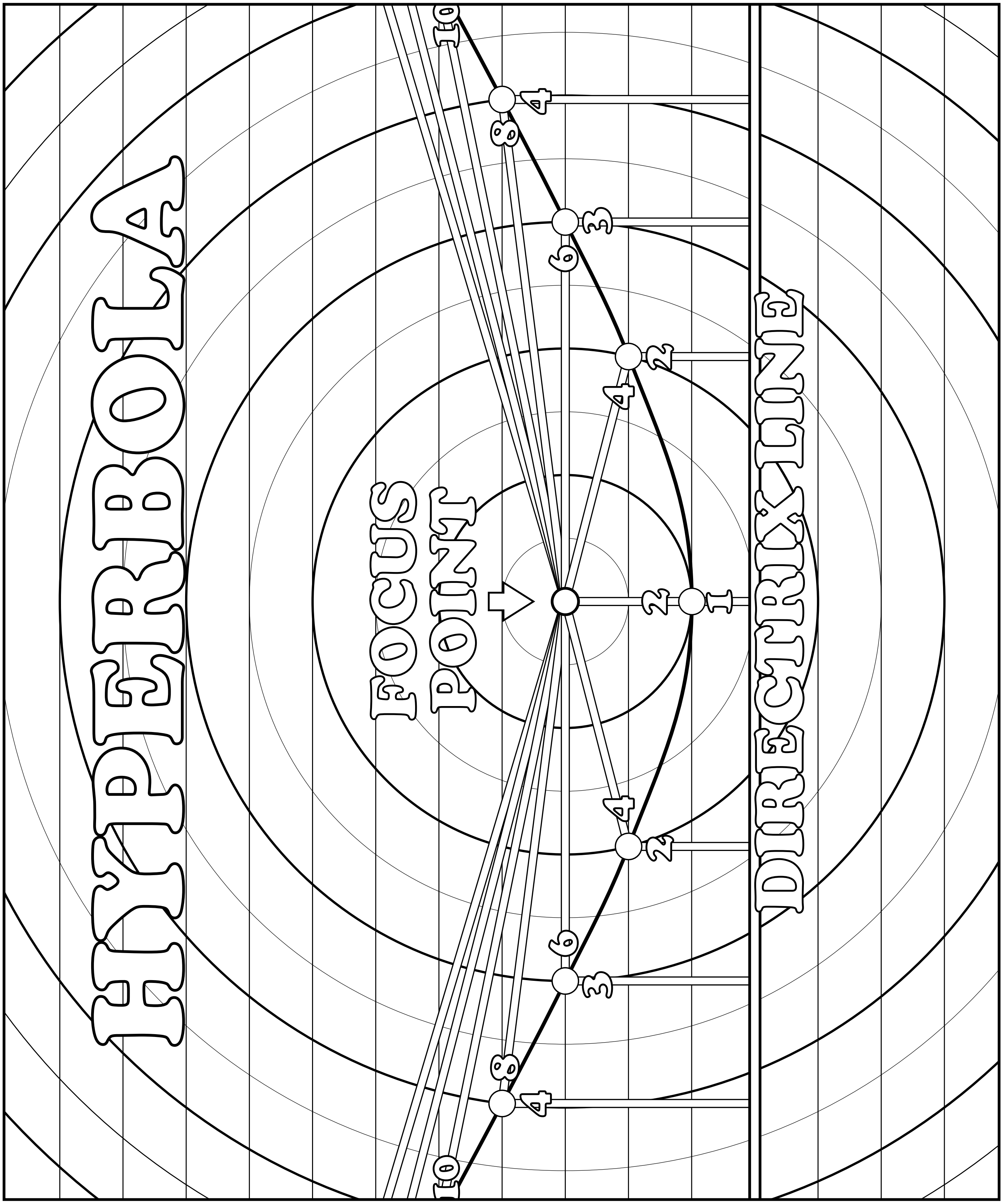
Evenly spaced concentric circles measure distance from a point.



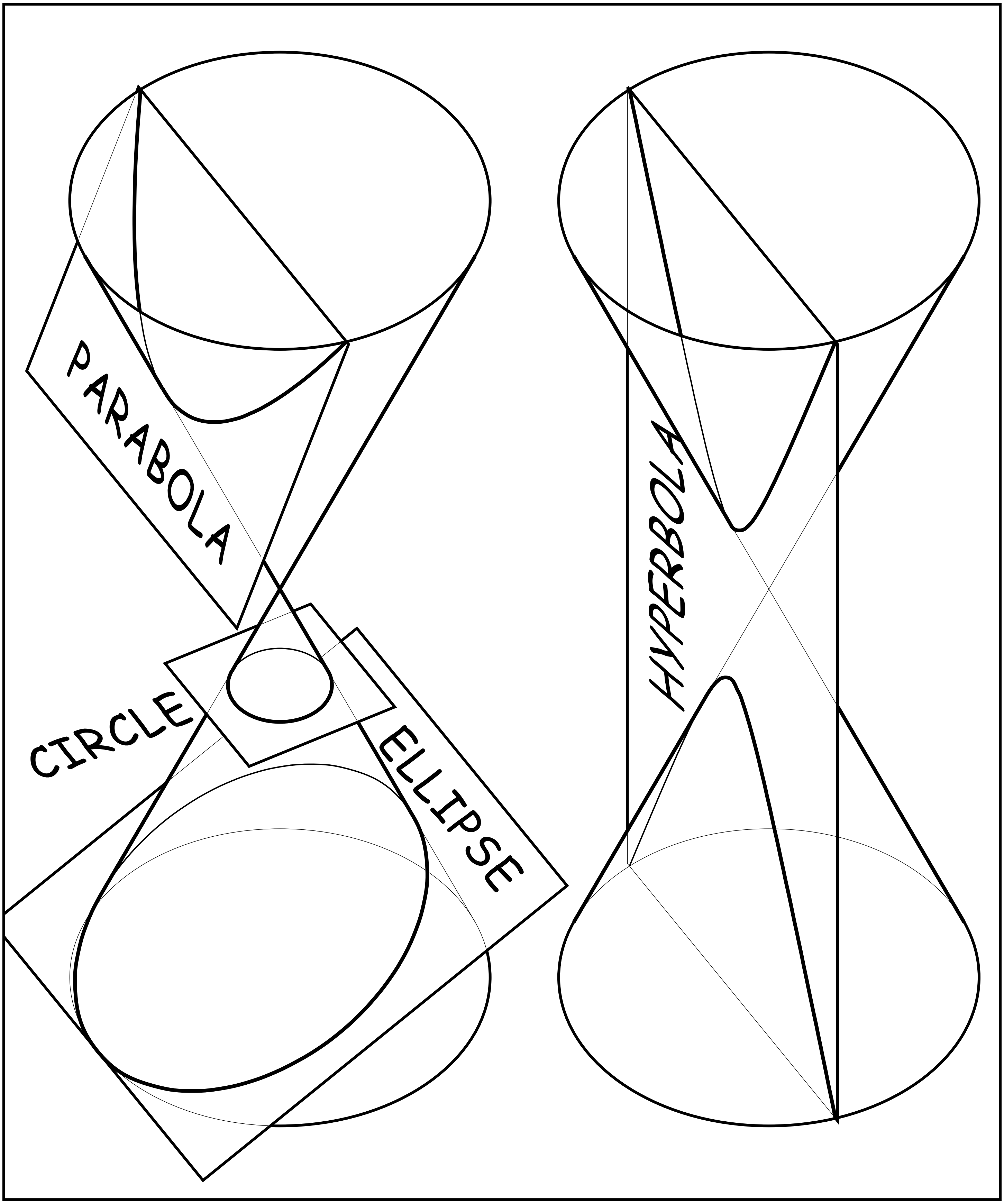
For each point on a parabola,
Distance to Focus Point = Distance to Directrix Line.
Eccentricity = 1.



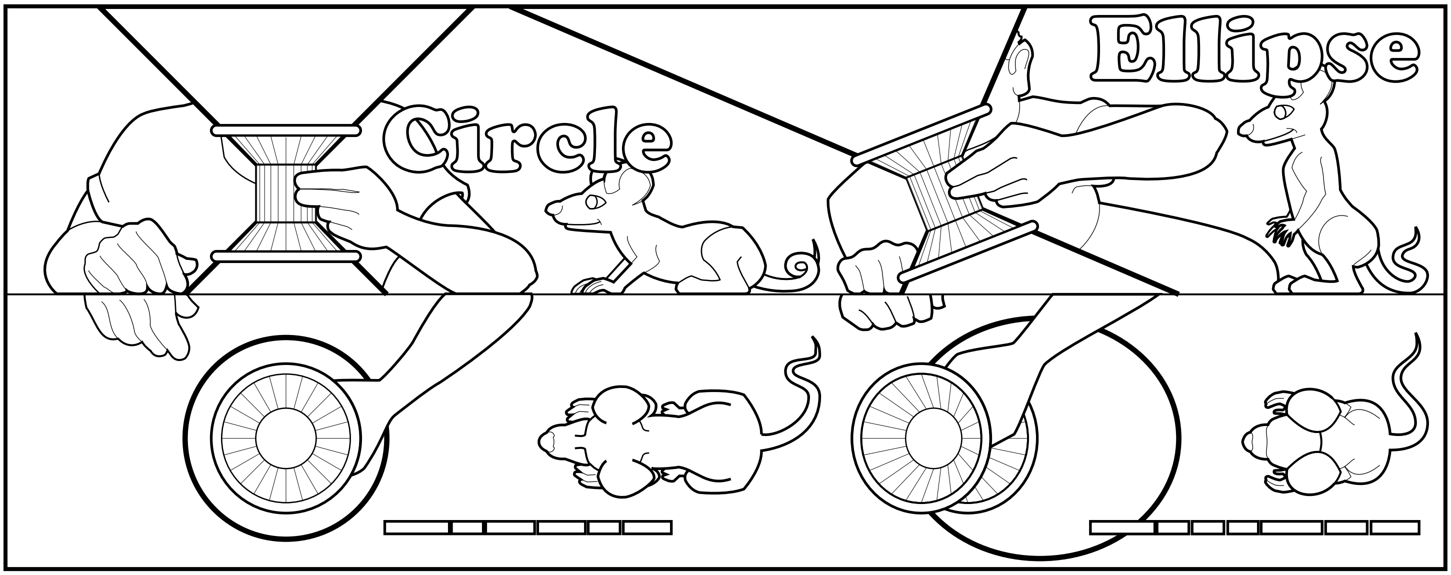
For each point on this ellipse,
Distance to Focus Point = 1/2 Distance to Directrix Line.
Eccentricity = 1/2.

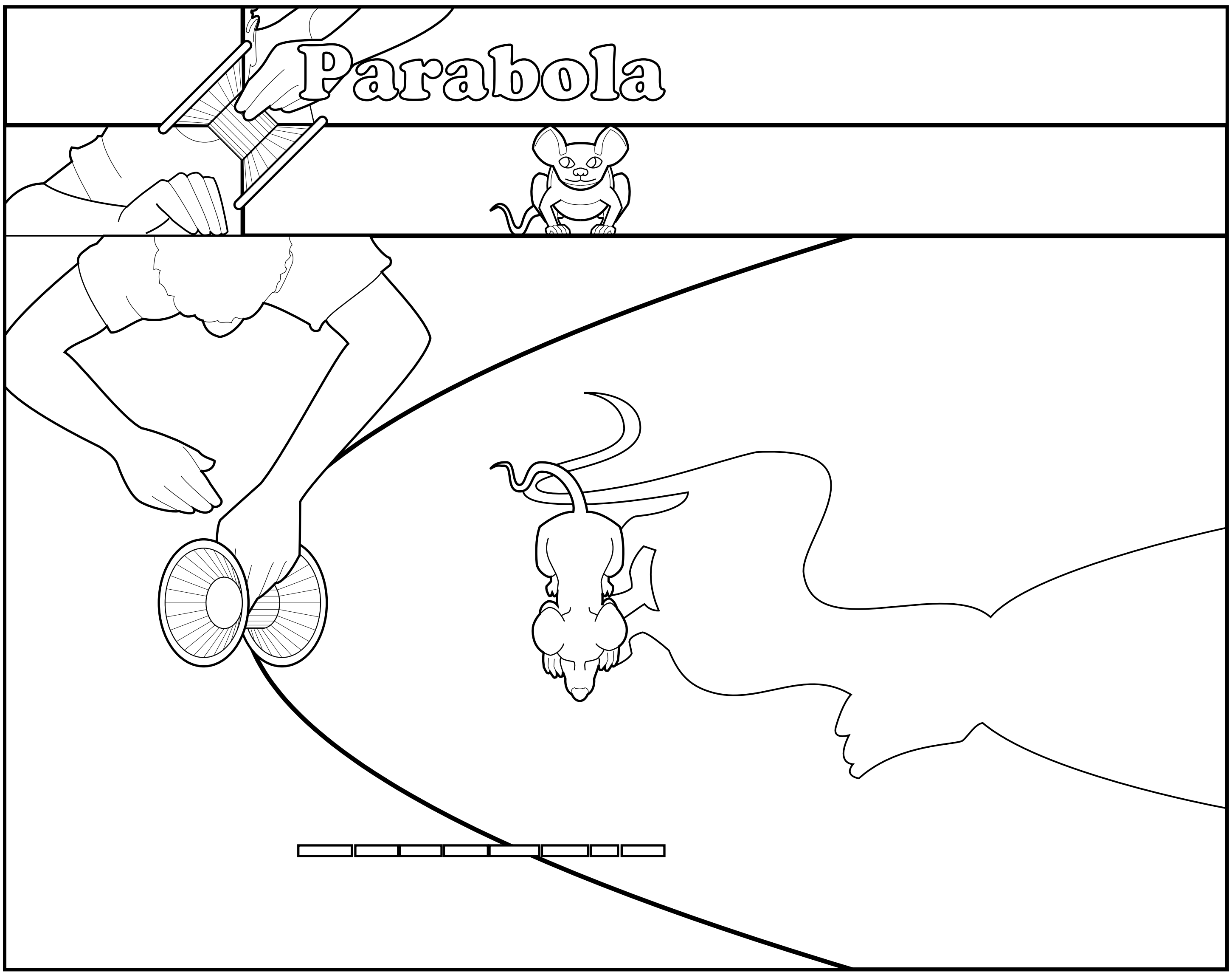


For each point on this hyperbola,
Distance to Focus Point = Twice Distance to Directrix Line.
Eccentricity = 2.



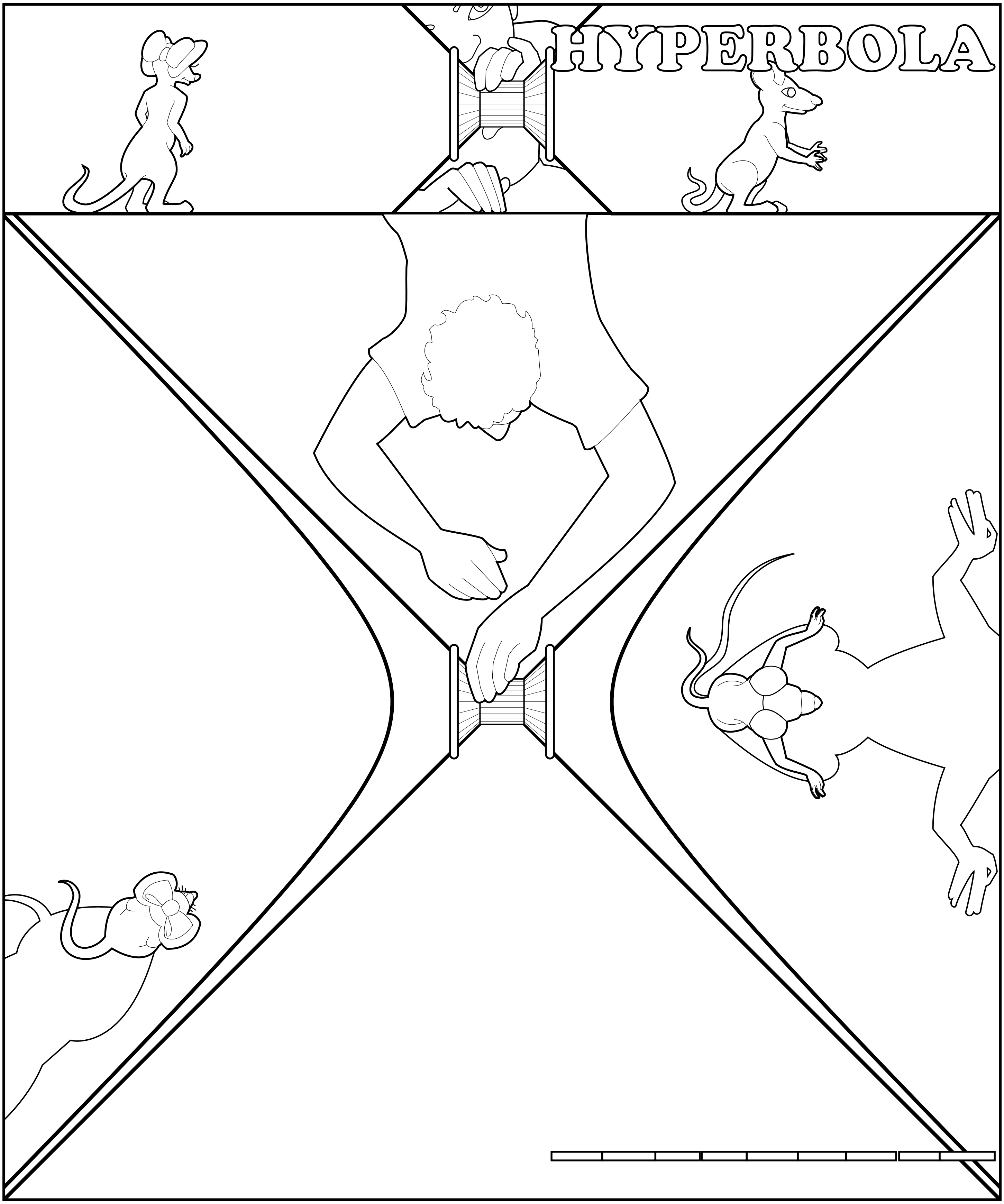
Conic sections come from cutting a cone with a plane.
The circle, ellipse, parabola and hyperbola
are all conic sections.



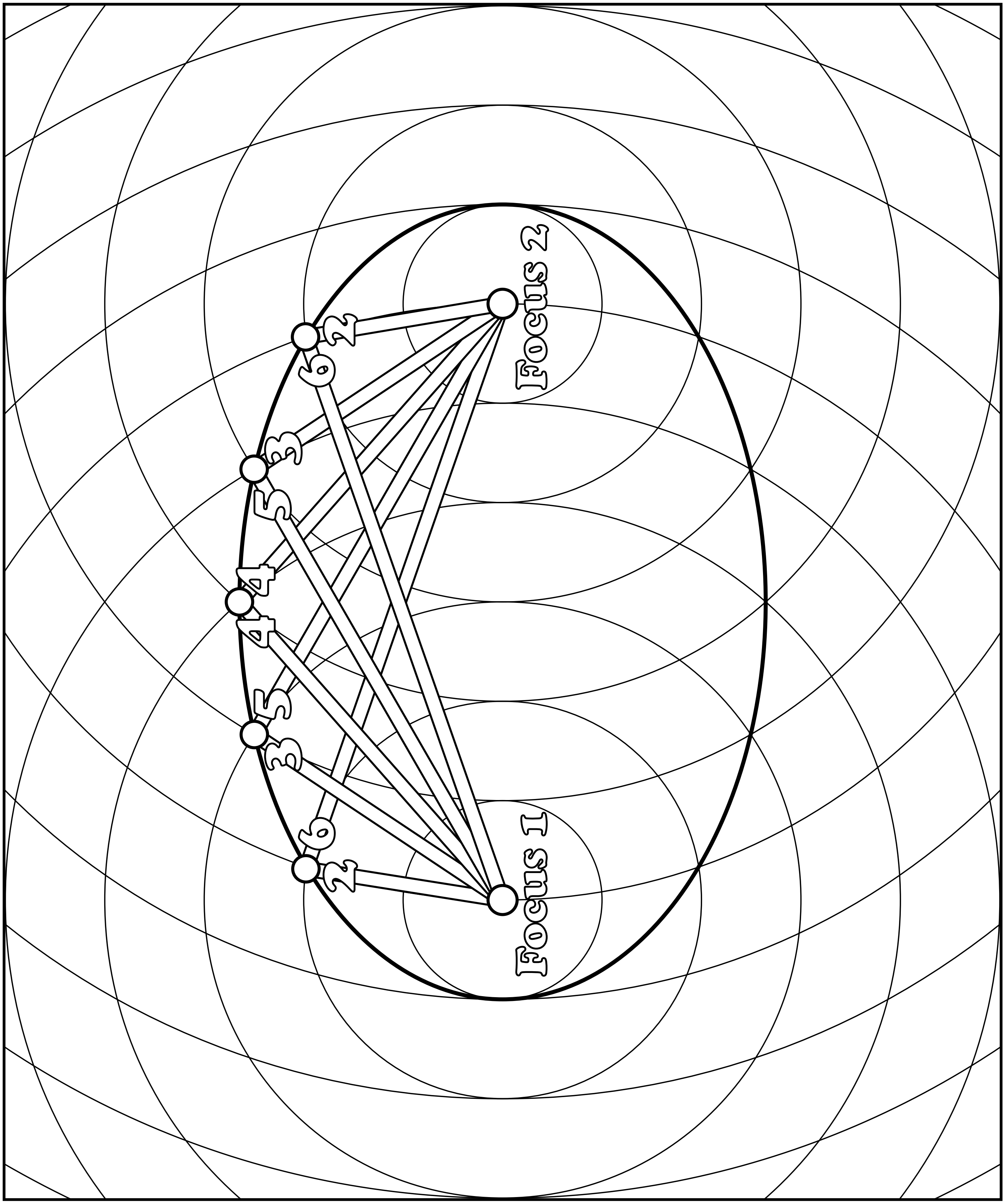


Conic Section means **Cut Cone**.

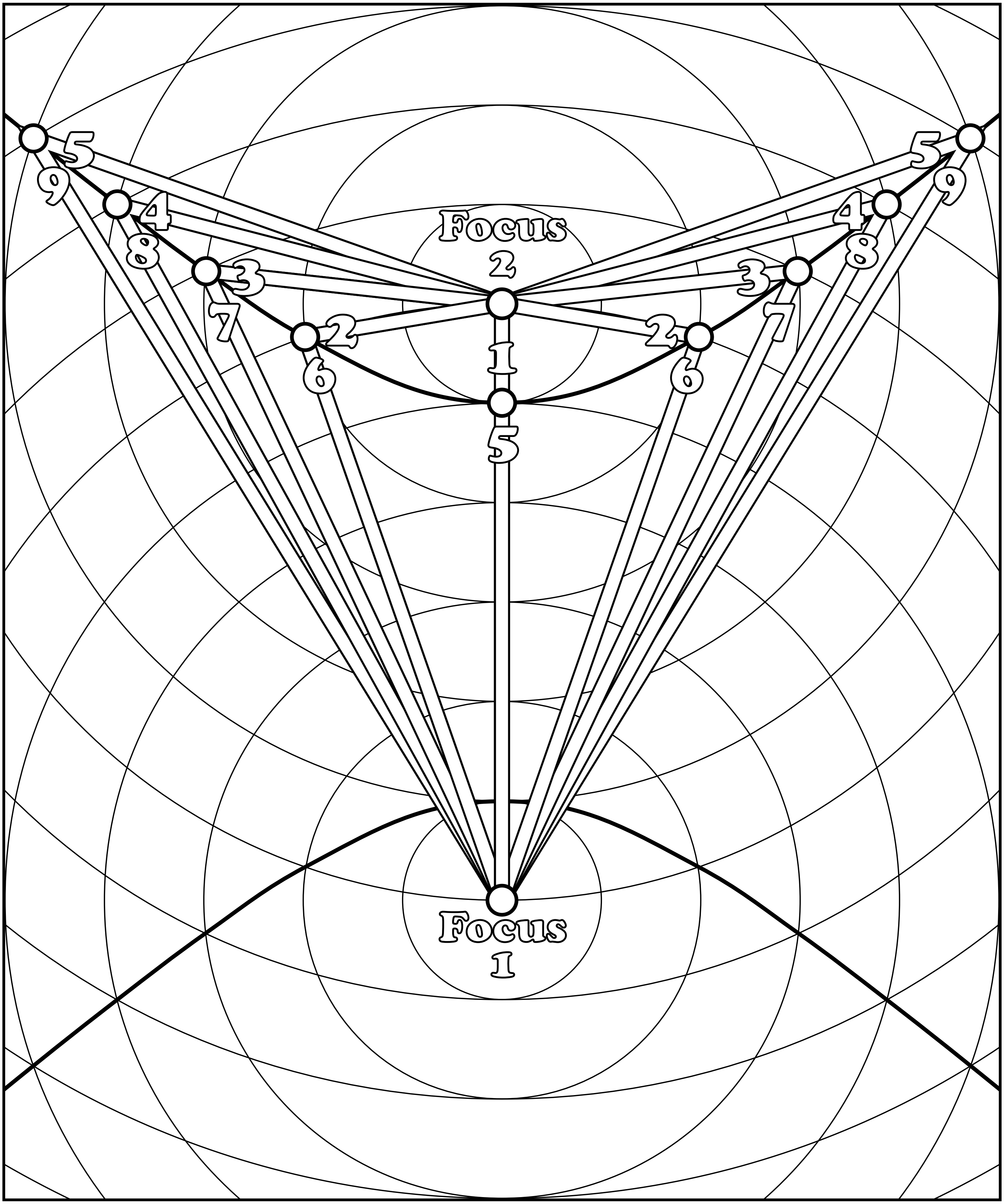
A flashlight beam is a cone and the floor is a plane that cuts it.
The circle, ellipse, parabola, and hyperbola are all conic sections.



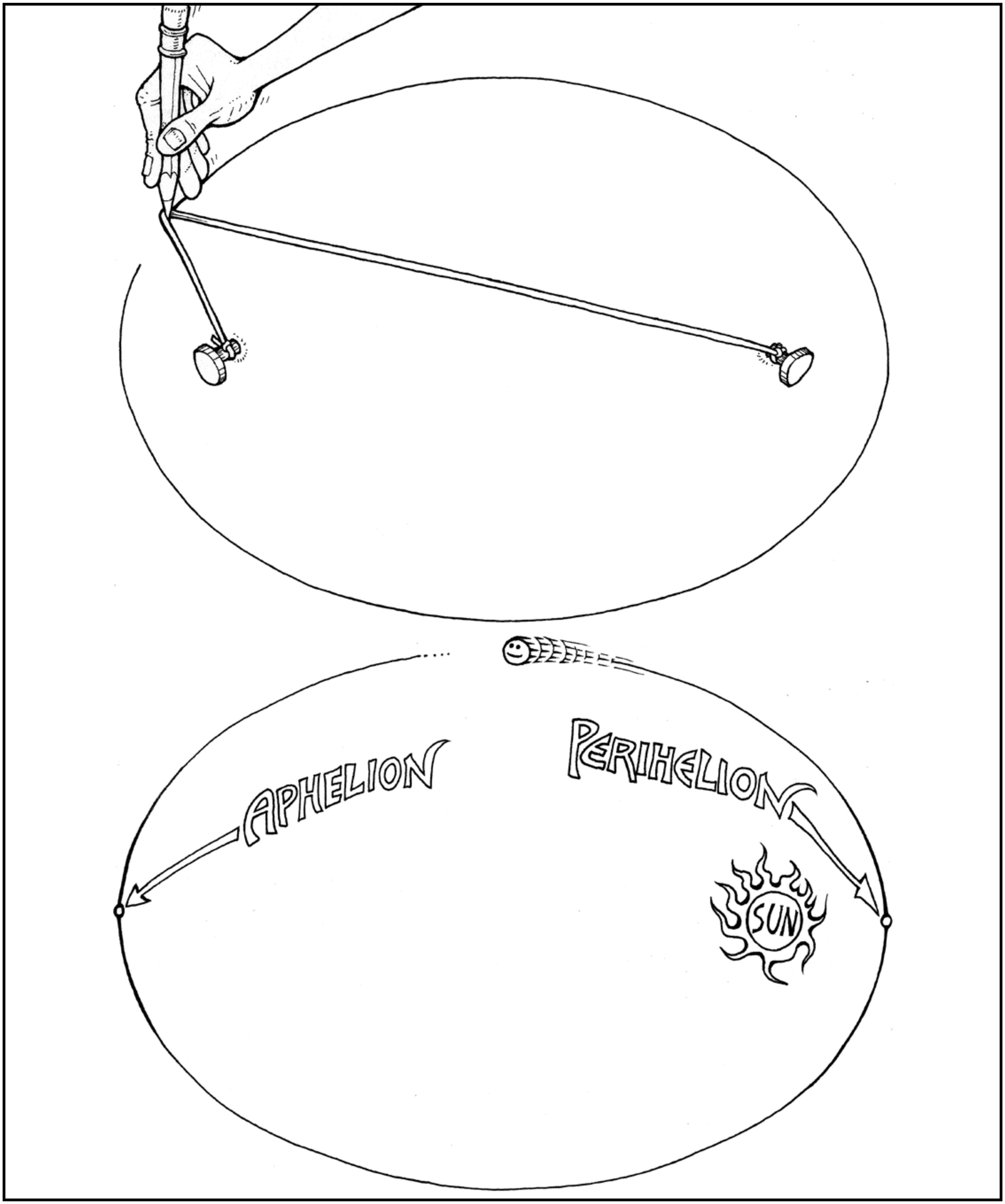
With a hyperbola the floor cuts both halves of the light cone. There are two lines the hyperbola gets closer and closer to but never touches. These are the hyperbola's **asymptotes**.



For each point on this ellipse,
Distance to Focus 1 + Distance to Focus 2 = 8.

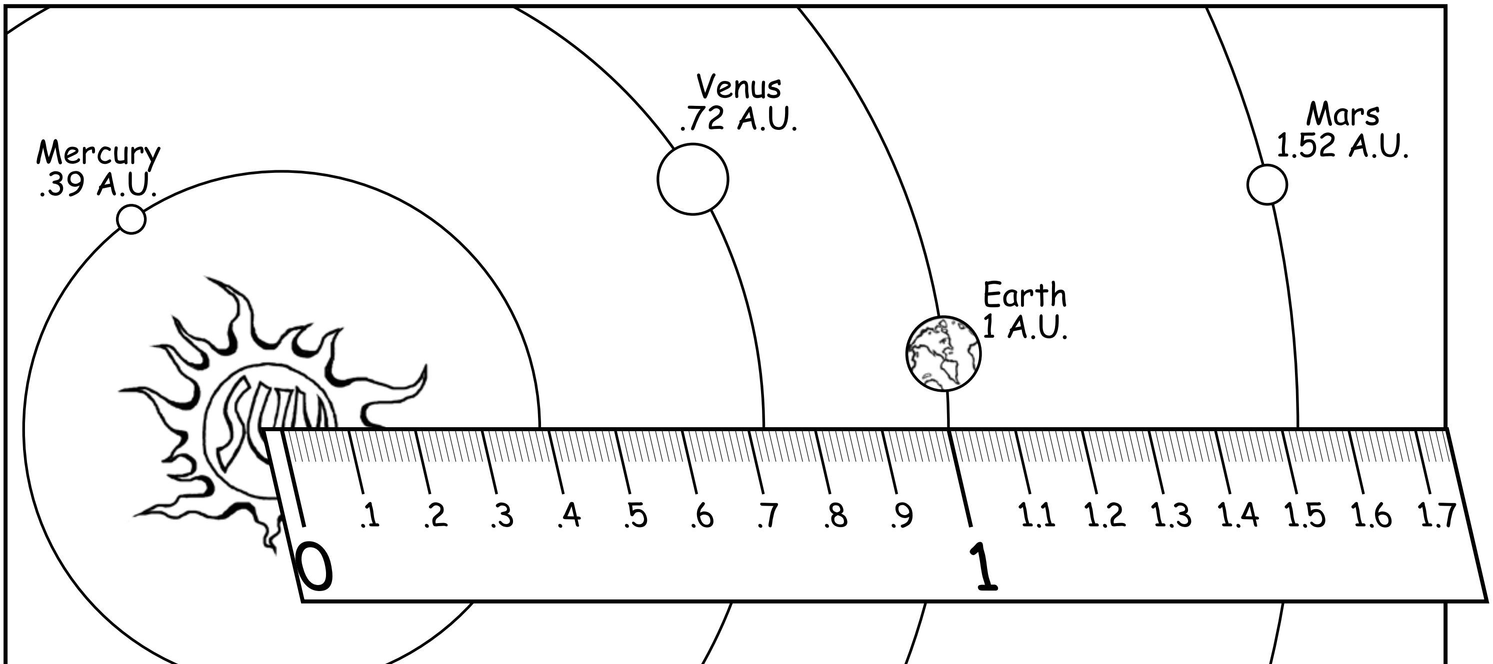


For each point on this hyperbola,
Distance to Focus 1 - Distance to Focus 2 = 4.

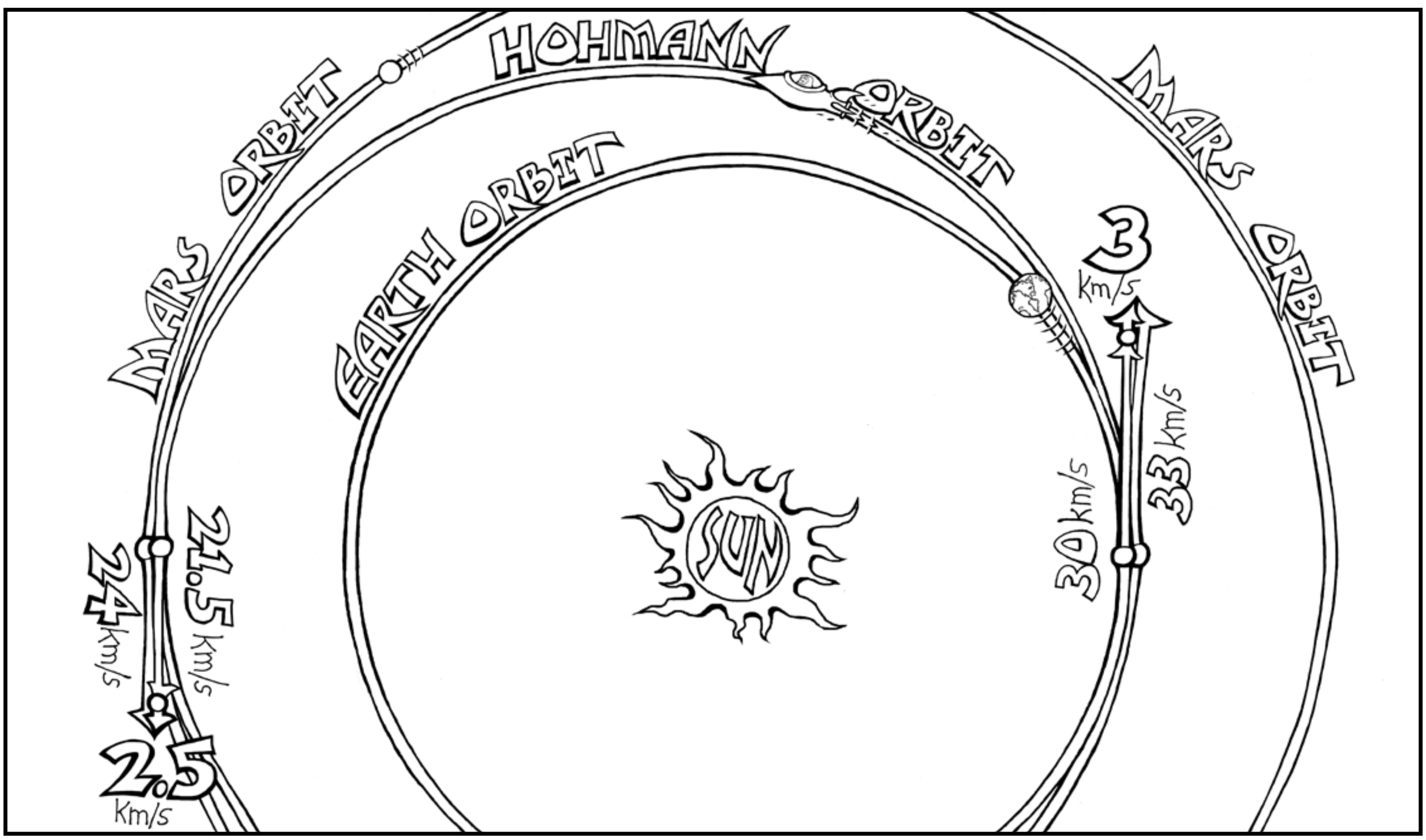


Tack two ends of a string to a sheet of drawing board.
Keeping the string taut, move the pencil. The path will be an ellipse with a tack at each focus.

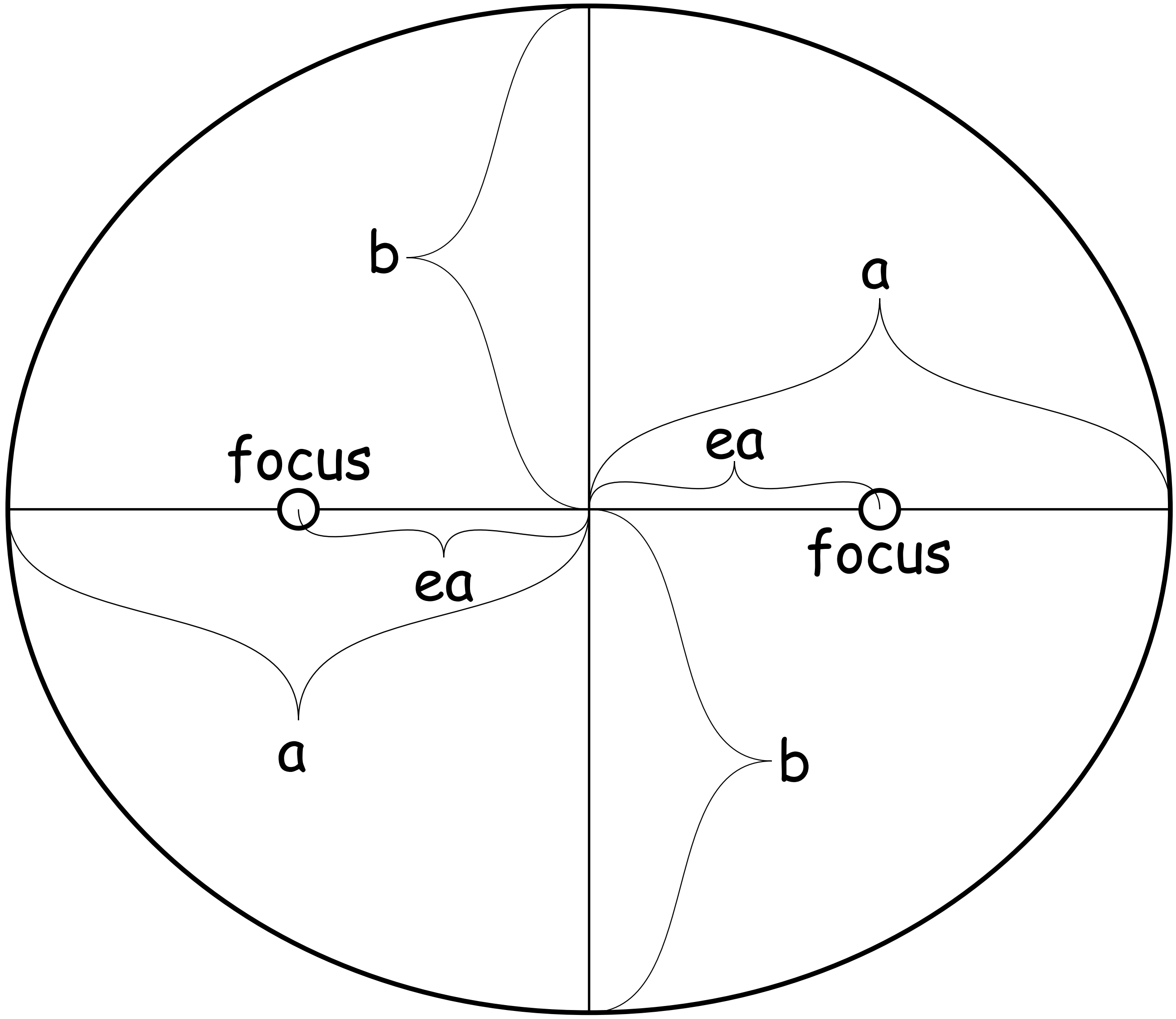
Planets, asteroids and comets move about our sun on ellipse shaped orbits.
The sun lies at one focus of the ellipse. This is **Kepler's First Law**.
The point closest to the sun is called the **perihelion**, the farthest point is the **aphelion**.



The average distance from earth's center to the sun's center is called an **astronomical unit**, or A.U. for short. Mercury's average distance from the sun is .39 A.U., Venus .72 A.U. and Mars average distance is 1.52 A.U.



A **Hohmann** orbit from earth to Mars is tangent to (just touches) the Earth orbit and Mars orbit. The Hohmann perihelion is at 1 A.U., the aphelion is at 1.52 A.U.
The earth moves around the sun at 30 kilometers/sec.
Mars moves around the sun at 24 kilometers a second.
At perihelion the space ship is moving 3 kilometers/second faster than earth.
At Aphelion, the spaceship is moving 2.5 kilometers/second slower than Mars.



Parts of an Ellipse

a = semi major axis

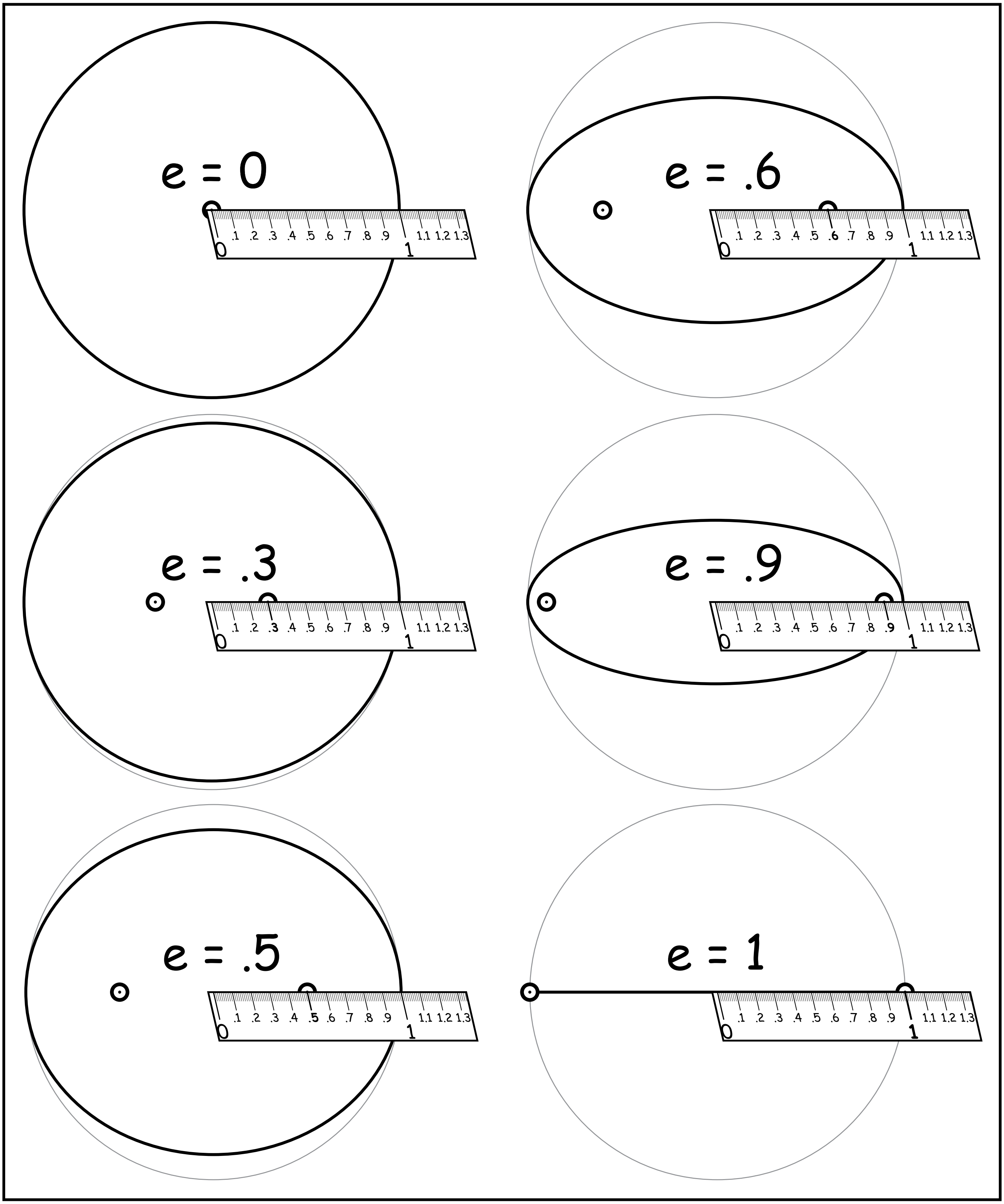
b = semi minor axis

e = eccentricity

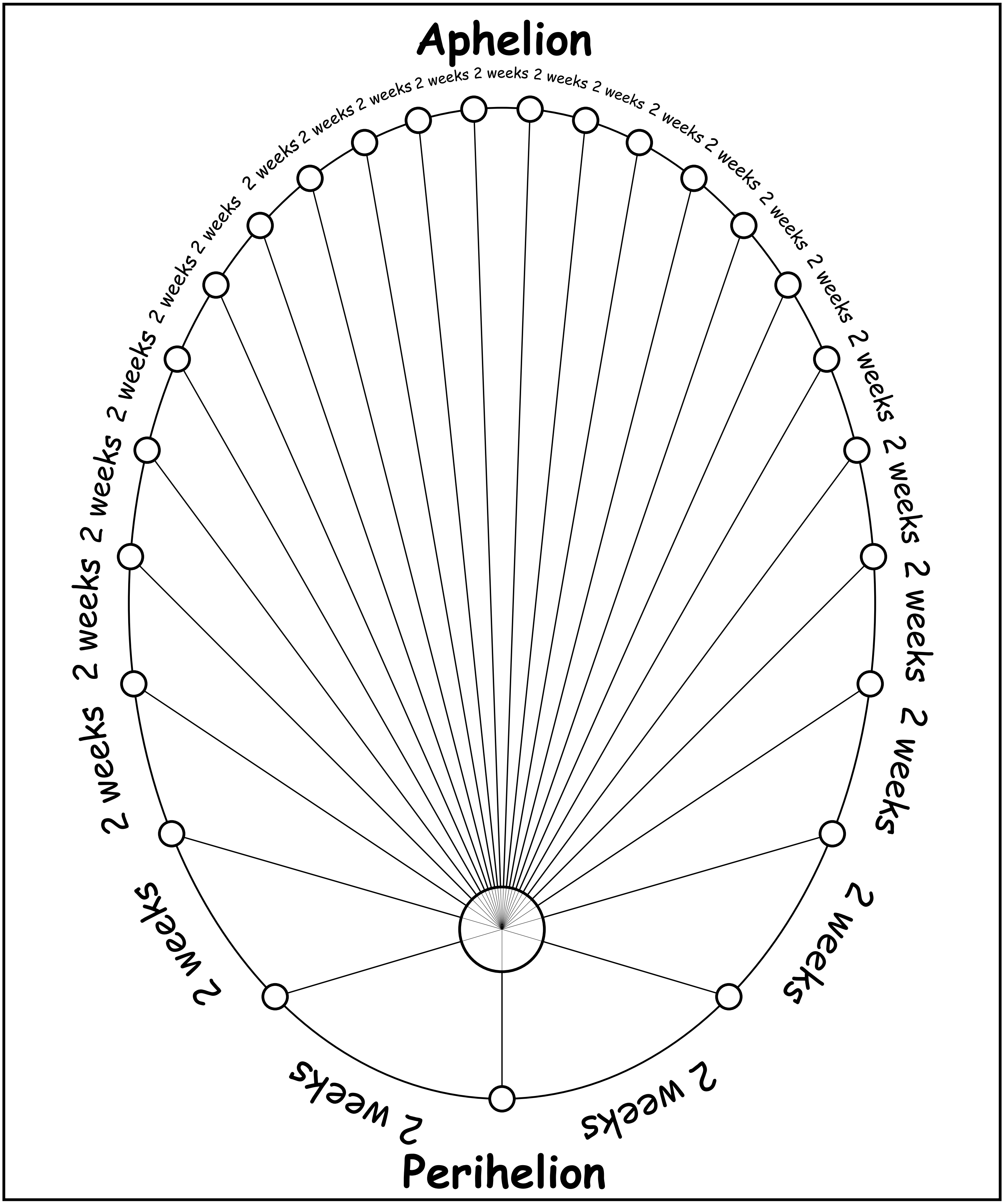
(in the above ellipse e = .5 or one half.)

ea = distance from ellipse center to focus

The semi major axis of an ellipse is often denoted with the letter **a**. The semi minor axis is usually called **b**. An ellipses' eccentricity is often labeled **e**.



In all of these ellipses a = 1. That is the semi major axis is one unit long.
The circle is a special ellipse of eccentricity zero.
As eccentricity gets closer to one, the foci move from the center to the edge.
A line segment could be regarded as an ellipse of eccentricity 1.

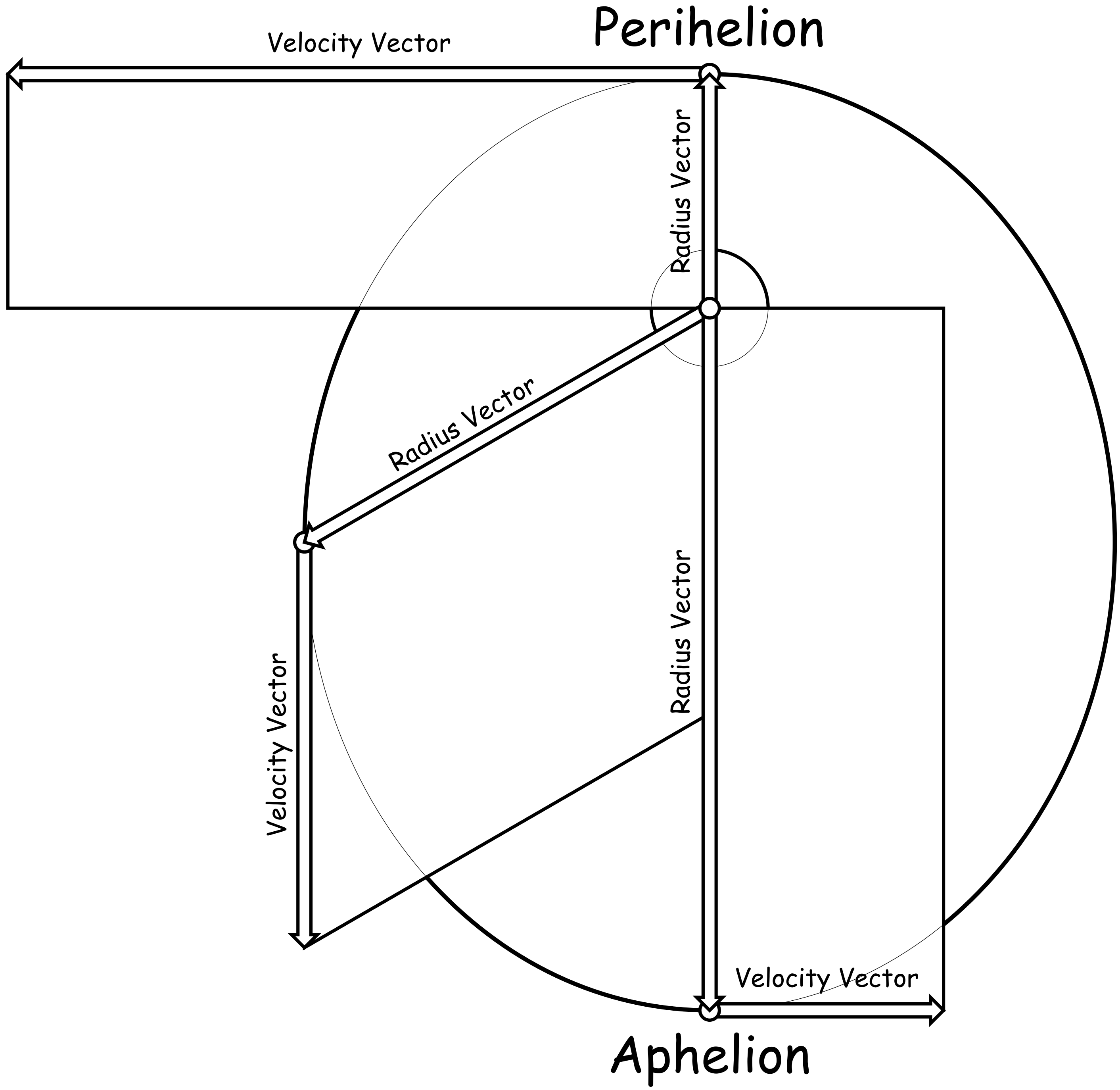


Over 2 weeks this orbit sweeps a wedge. Some wedges are short & fat, others tall & skinny.

But they all have the same area.

An orbiting body sweeps equal areas in equal times.

This is **Kepler's Second Law**.

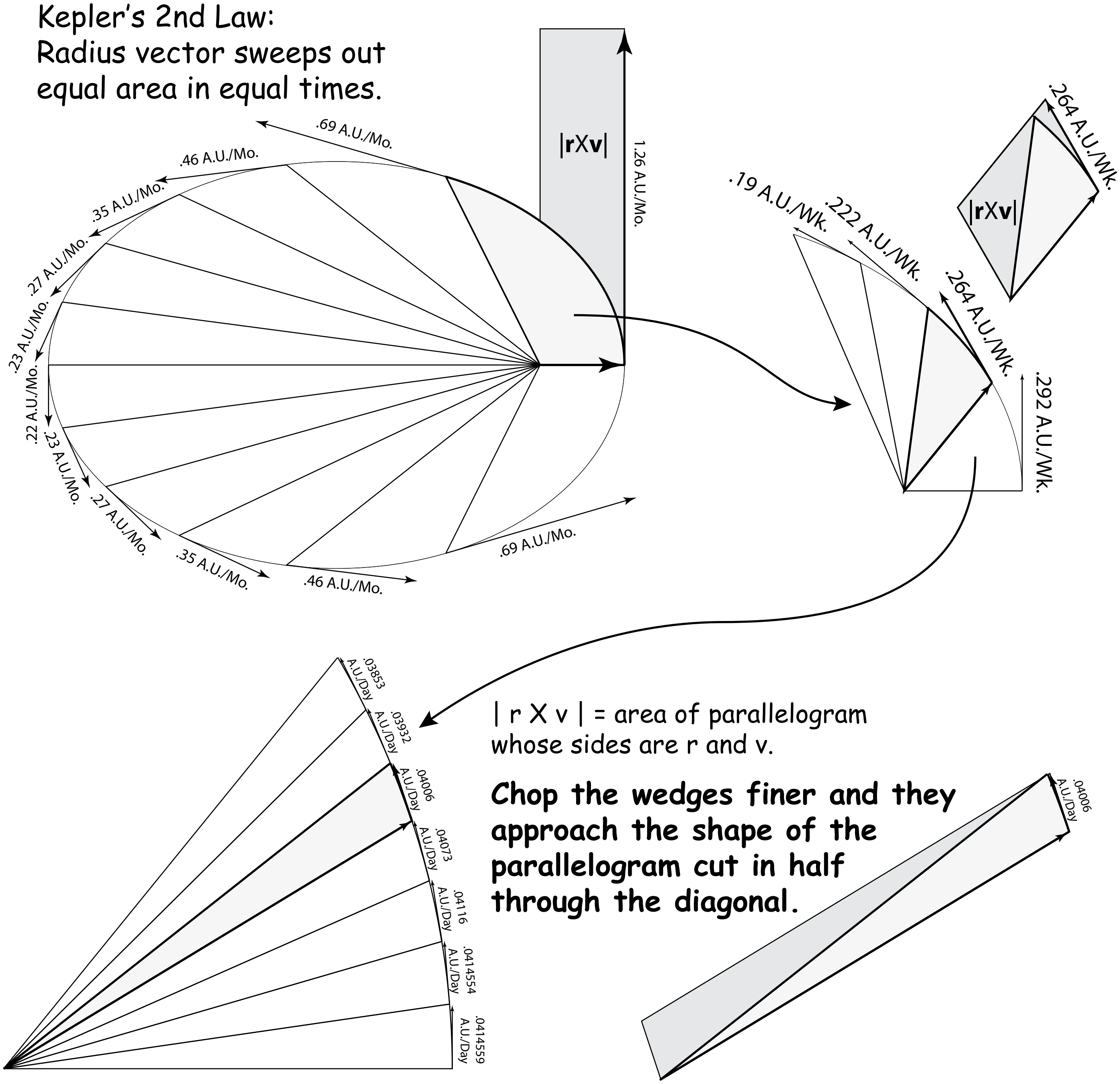


Radius Vector X Velocity Vector
=
Specific Angular Momentum

When dealing with vectors an **X** denotes **cross product**.
Magnitude of a cross product can be thought of as area of the parallegram.

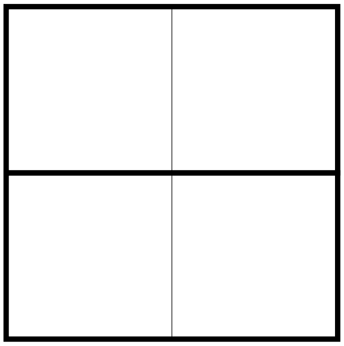
The two rectangles and parallelogram pictured above all have the same area.

As an object gets closer to the sun it goes faster and the velocity vector gets bigger. The Radius Vector and velocity vector make two sides of parallelogram. The area of the parallelogram stays the same. At perihelion and aphelion the parallelogram is a rectangle.



Cross product of position and velocity vectors is twice the area the vector sweeps out in a given time.

Chopping into finer wedges it becomes obvious | **r** X **v** | is twice the area of a wedge swept out over a given time. Summing all the wedges we can see **specific angular momentum is twice (area of the ellipse)/(orbital period).**

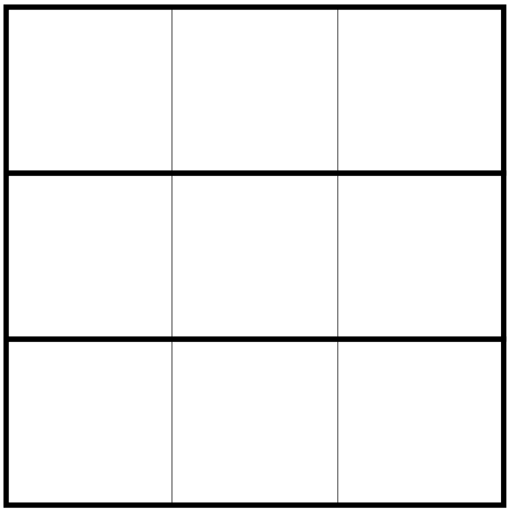


$$2^2 = 2 \times 2 = 4$$

2 squared is 2 times 2 which is 4.
Another way to read it:
2 to the second power equals 2 times 2 which equals 4.
Can you see why **2 to the second power** is also called **2 squared**?

$$4^{1/2} = 2$$

4 to the half power is 2. Or: The square root of 4 is 2.

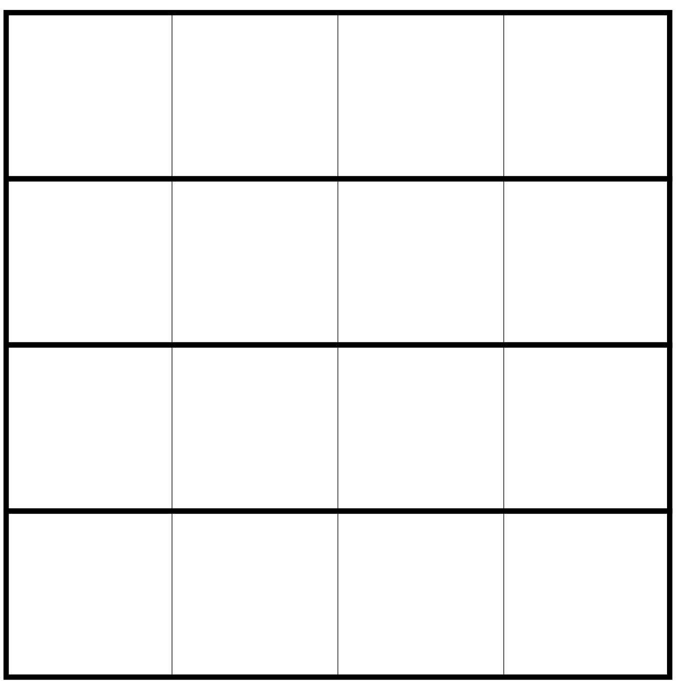


$$3^2 = 3 \times 3 = 9$$

3 squared is 3 times 3 which is 9.
Another way to read it:
3 to the second power equals 3 times 3 which equals 9.

$$9^{1/2} = 3$$

9 to the half power 3. Or: The square root of 9 is 3.



$$4^2 = 4 \times 4 = 16$$

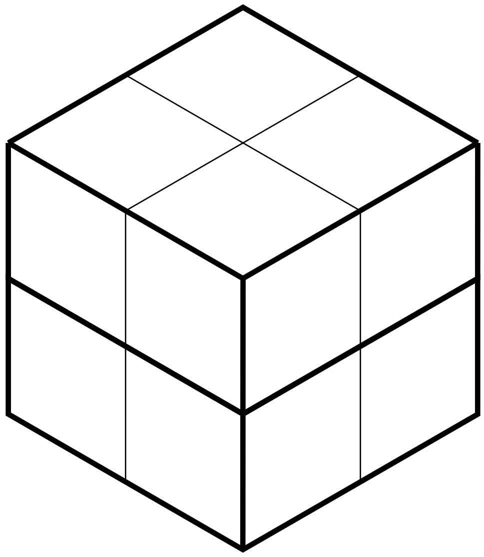
4 squared is 4 times 4 which is 16.

$$16^{1/2} = 4$$

16 to the half power is 4.
Or:
The square root of 16 is 4.

Squares and Square Roots

This may not seem related to conic sections and orbital mechanics.
But we will use these concepts in Kepler's Third Law.

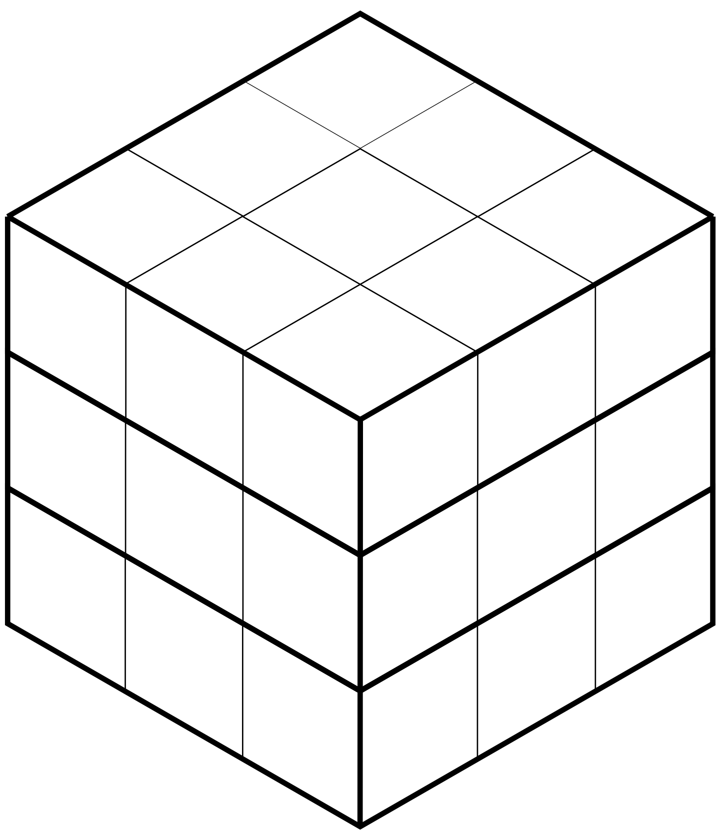


$$2^3 =$$
$$2 \times 2 \times 2 =$$
$$2 \times 4 =$$
$$8$$

2 to the third power is 8.
or:
2 cubed is 8.

$$8^{1/3} = 2$$

8 to the one third power is 2.
Or: The cube root of 8 is 2.



$$3^3 =$$
$$3 \times 3 \times 3 =$$
$$3 \times 9 =$$
$$27$$

3 to the third power is 27.
or:
3 cubed is is 27.

$$27^{1/3} = 3$$

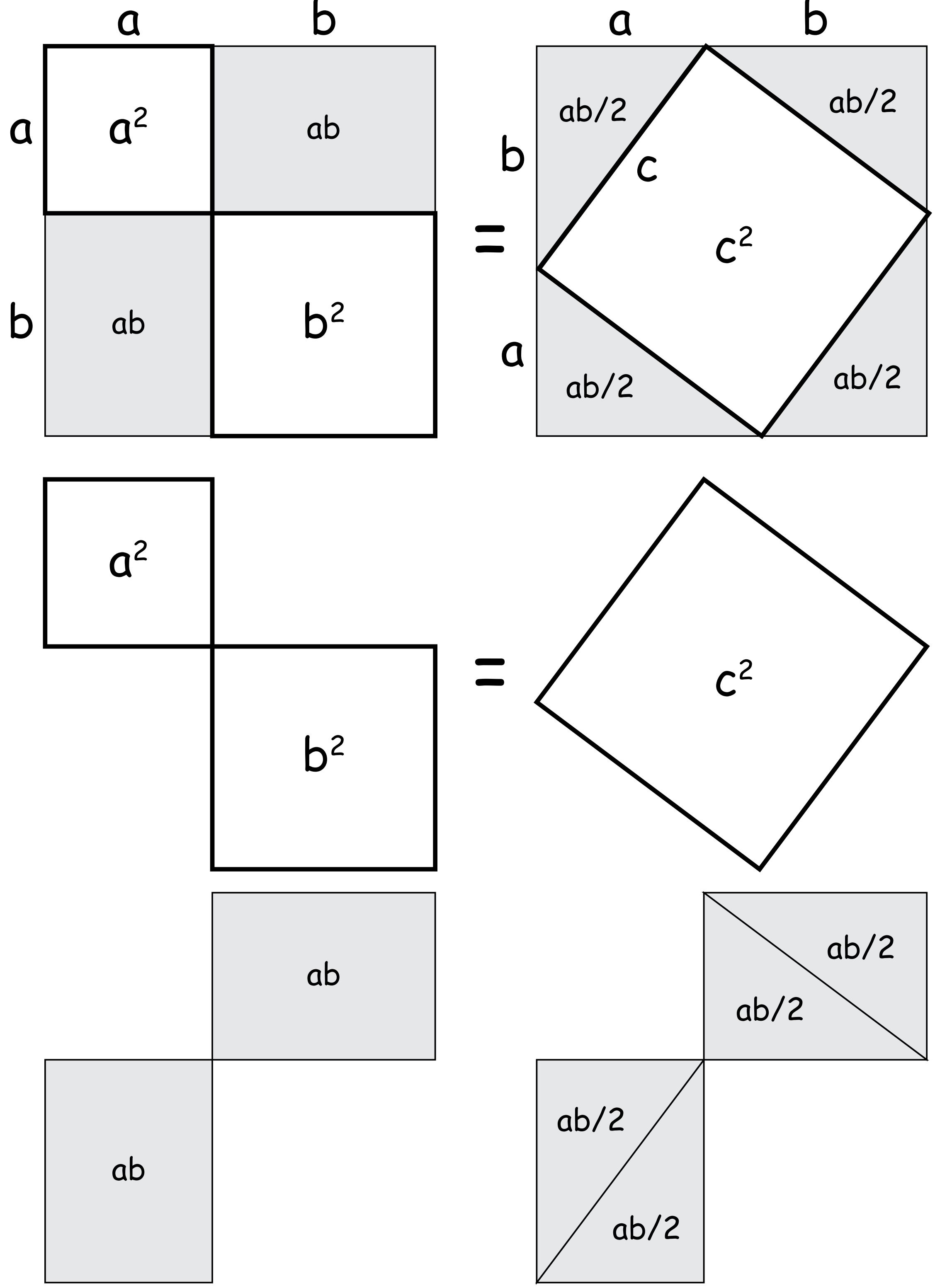
27 to the one third power is 3.
The cube root of 27 is 3.

Cubes and Cube Roots

These are also concepts used in Kepler's Third Law.

PYTHAGOREAN THEOREM

Both squares have side lengths (a+b)
So the both have the same area: $(a+b)^2$

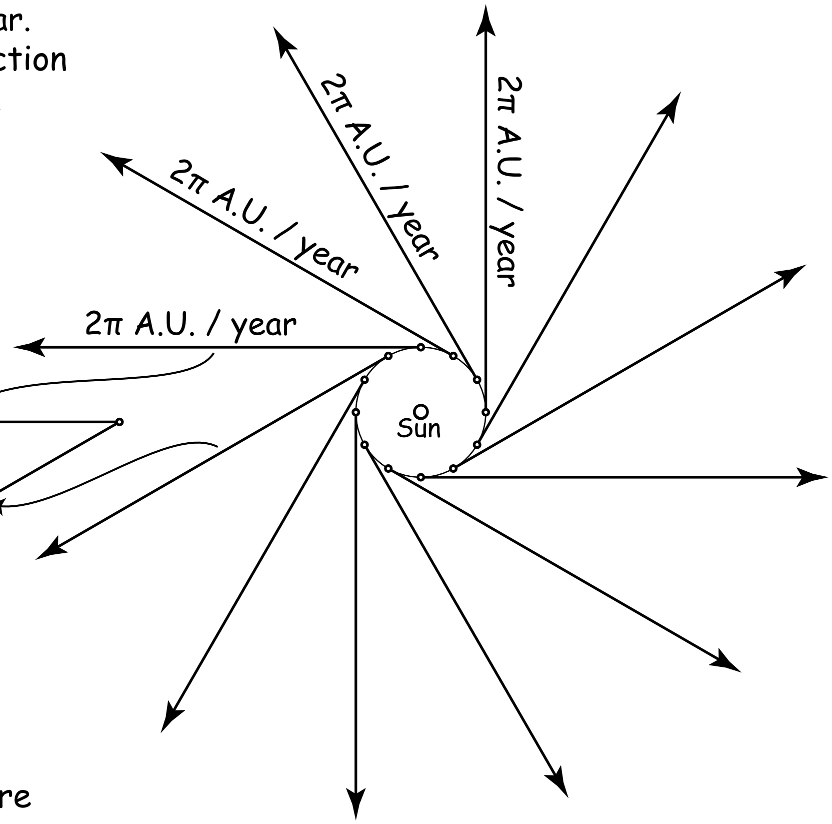


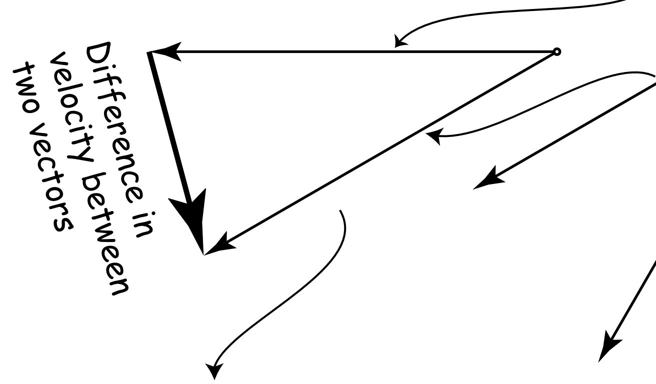
Given a right triangle with legs a and b, and hypotenuse c,

$$\mathbf{a^2 + b^2 = c^2}$$

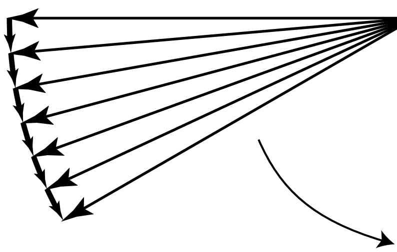
Earth is moving about 2π A.U./year.
The velocity vector changes direction during the circuit around the sun.

To get change of velocity from one month to the next, place the foot of one vector on the foot of another. The vector from one tip to the other is the change.





Between these two vectors there are many intermediate vectors.



Over a year's time the velocity vector traces a circle of circumference

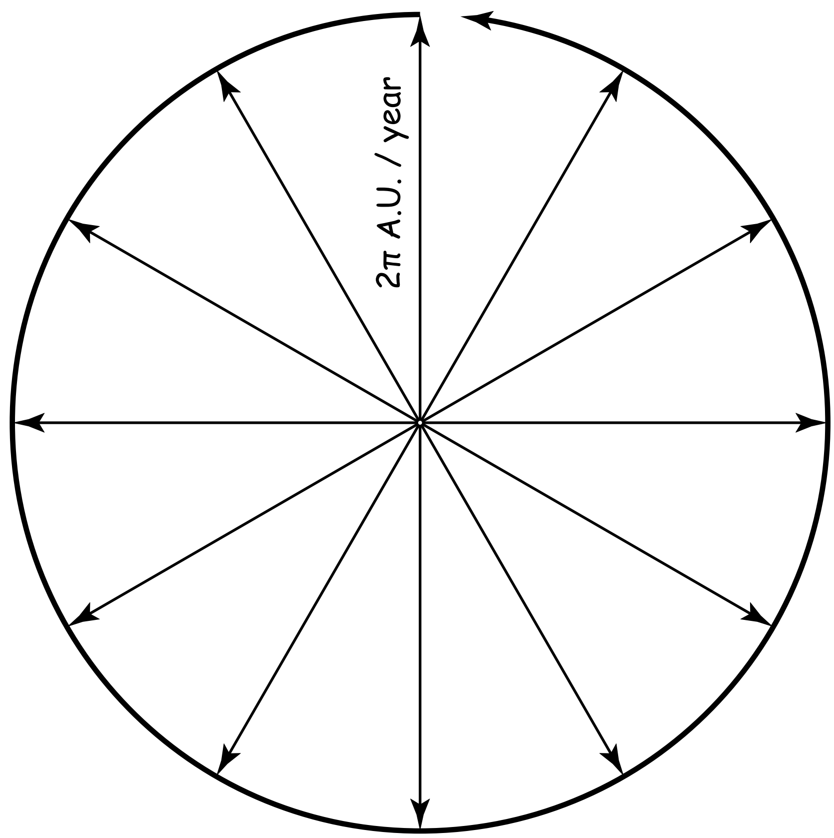
$2\pi * v$

or

$2\pi/\text{year} * 2\pi$ A.U. / year

$= 2\pi^2/\text{year}^2 *$A.U.

$= \omega^2 r$



Centrifugal Acceleration = $\omega^2 r$

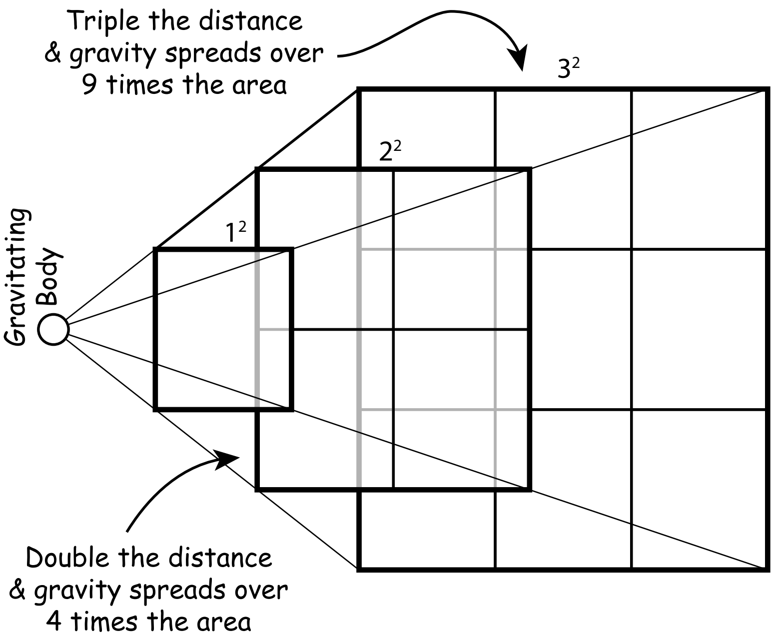
Christiaan Huygens showed this in 1859

Calling the period of a circular orbit T, (2π radians /T) is ω, the angular velocity.
Circle radius = r.

Centrifugal acceleration is $\omega^2 r$.

So centrifugal acceleration is $\omega^2 r$.

The so-called centrifugal force isn't really a force but inertia in a rotating frame.



Gravity falls off with inverse square of distance.
Gravity acceleration = GM / r^2.

G is the universal gravitational constant
M is the mass of the gravitating body and
r is the distance of the body.

In a circular orbit the orbiting body stays the same distance from the central gravitating body. Force of gravity cancels centrifugal force

So we can say

$GM / r^2 = \omega^2 r$

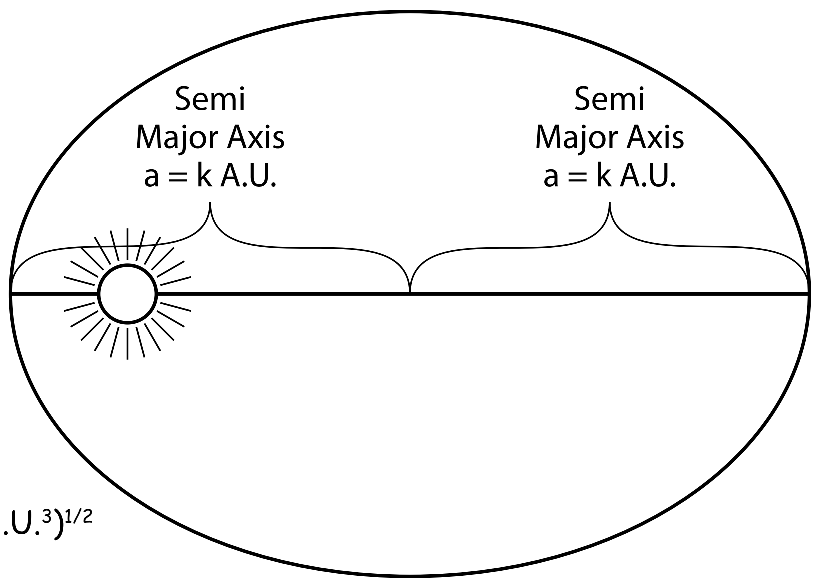
$GM = \omega^2 r^3$

$$GM = \omega^2 r^3$$

In the case of earth's orbit about the sun, we see

$$GM = (2\pi / \text{Year})^2 * \text{A.U.}^3 .$$

Kepler's Third Law



Orbital Period T is given by

$T = 2\pi (a^3 / GM)^{1/2}$

Where a = k A.U..

Substitute
$(2\pi / \text{Year})^2 * \text{A.U.}^3$ for GM
and k A.U. for a,

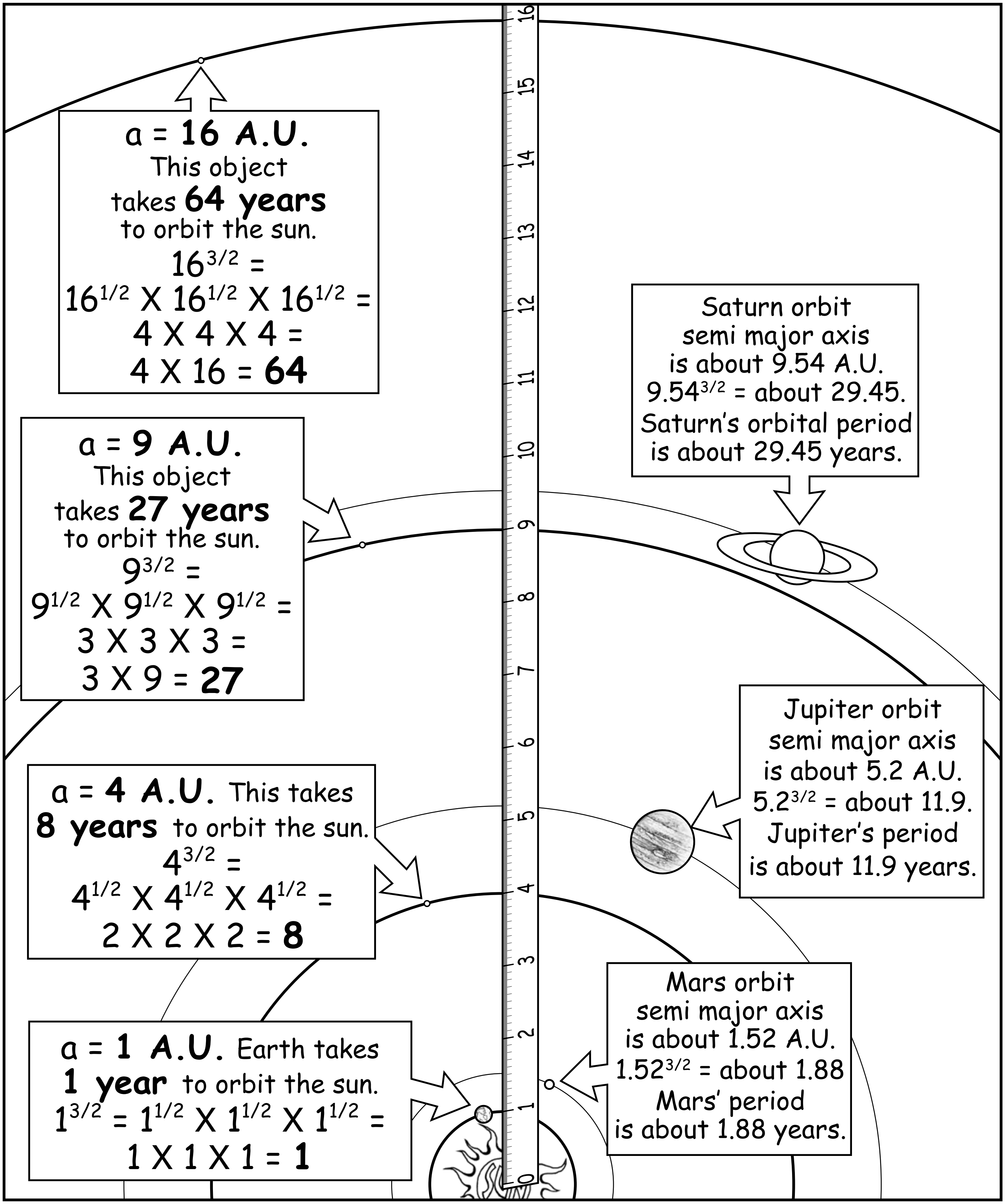
$T = 2\pi ((k\ \text{A.U.})^3 / ((2\pi / \text{Year})^2 * \text{A.U.}^3)^{1/2}$

$T = 2\pi (k^3 * (\text{Year} / 2\pi)^2)^{1/2}$

$T = k^{3/2}$ Years

$$T = k^{3/2} \text{ Years}$$

Kepler's Third Law:
Orbital period is proportional to
length of semi major axis raised to 3/2 power.

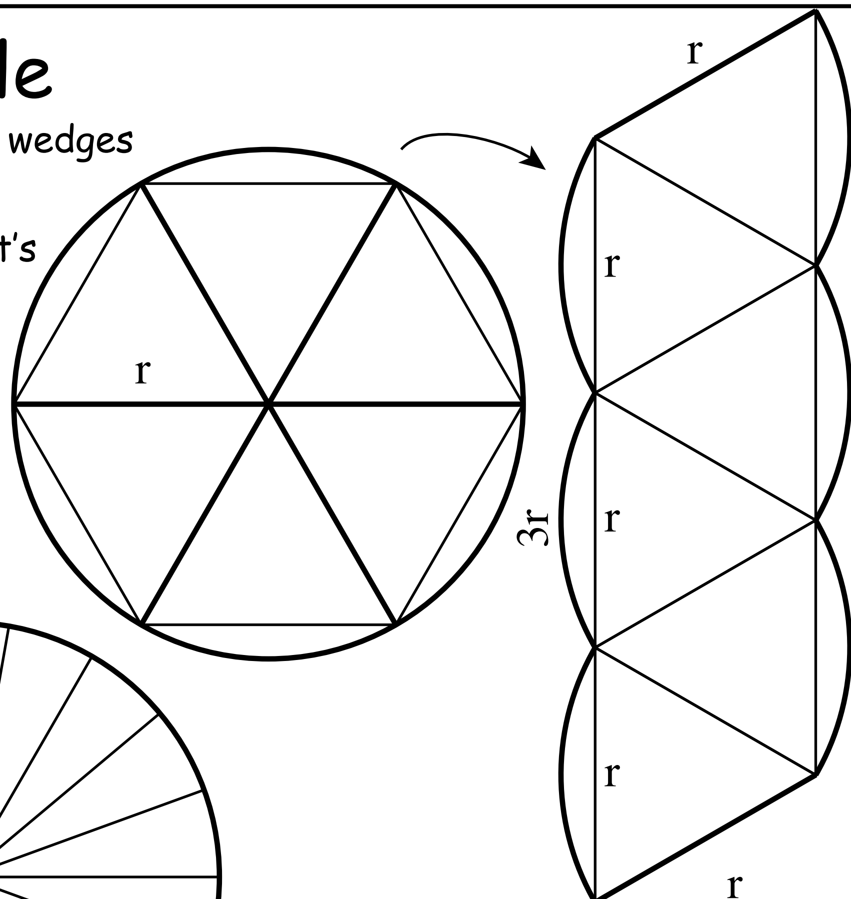


The number of astronomical units of the semi-major axis raised to the 3/2 power gives the number of years a body takes to orbit the sun. This comes from **Kepler's Third Law**.

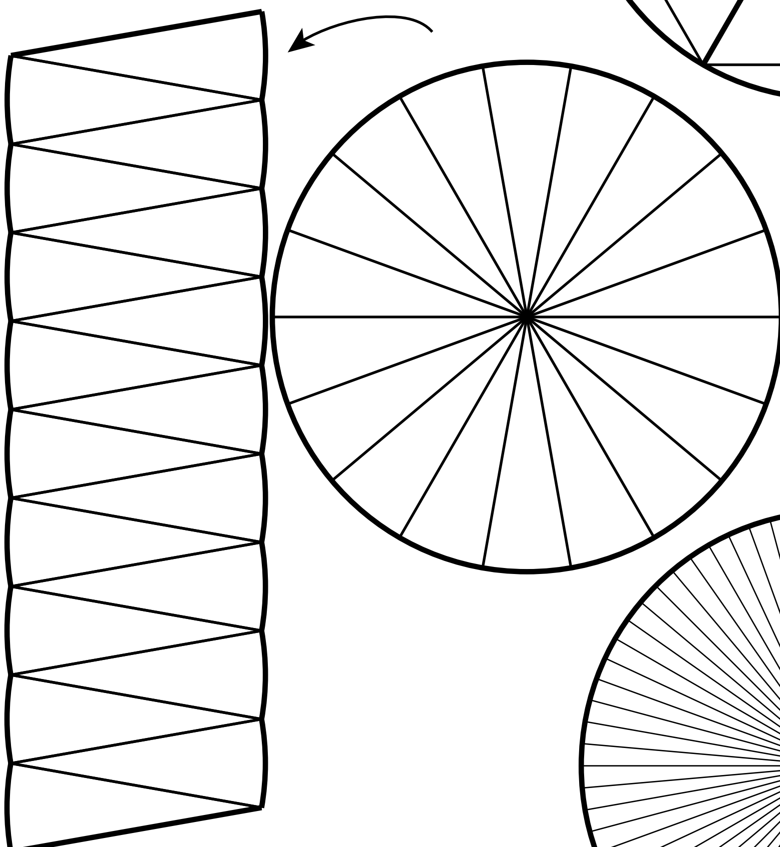
Area Of A Circle

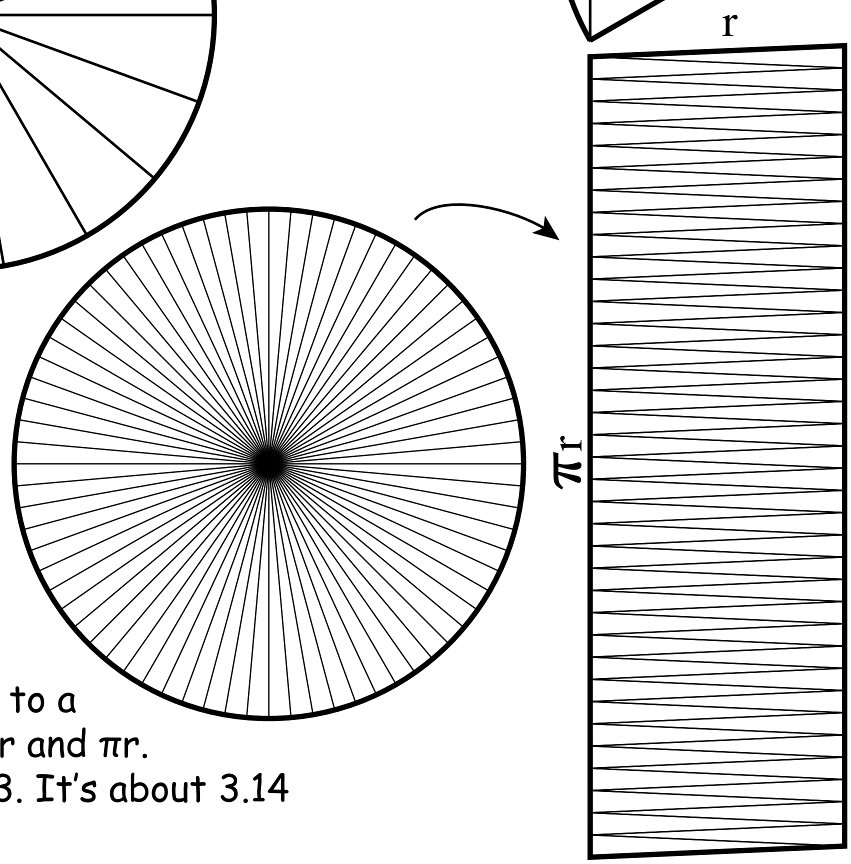
Slice a circle into six wedges and re-arrange.

You have a shape that's a bit more than a parallelogram with sides 3r by r.



Slice the circle into finer wedges and re-arrange.



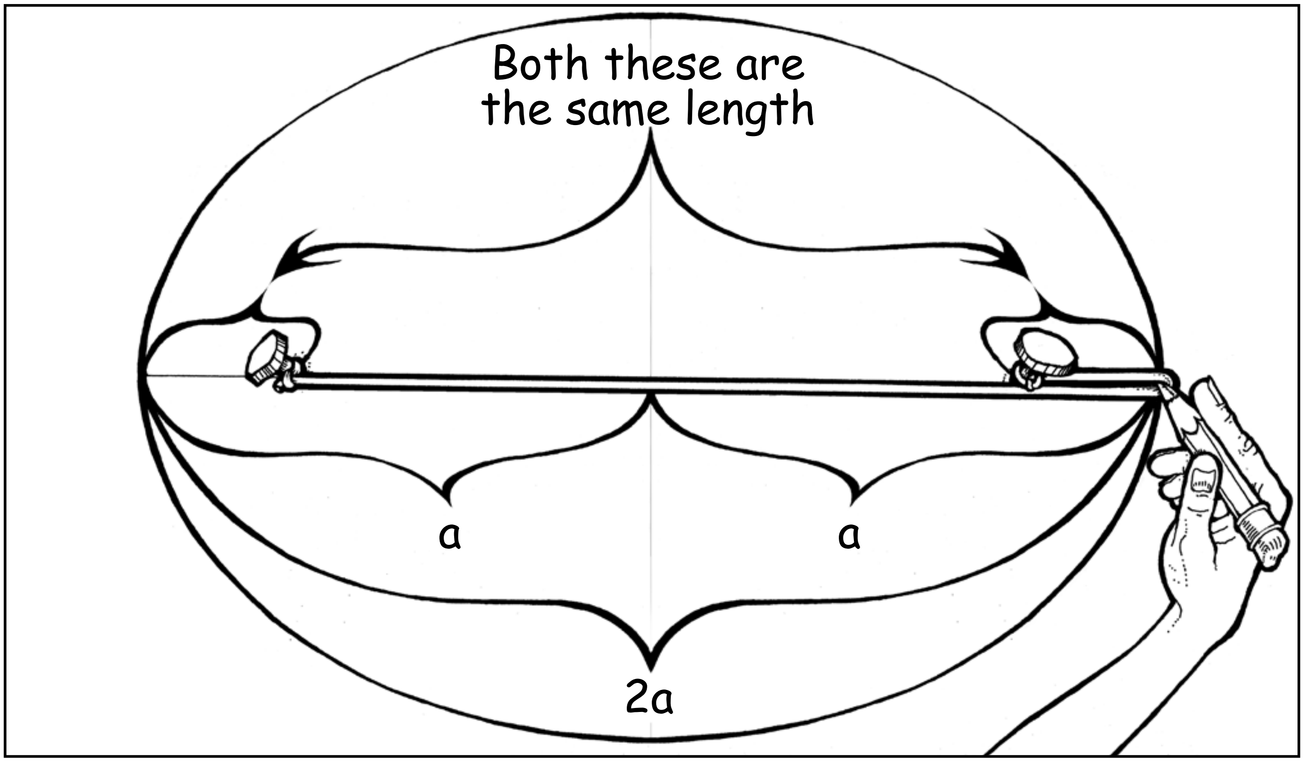


The finer the wedges,
the closer the circle is to a
rectangle having sides r and πr.
π is a little more than 3. It's about 3.14

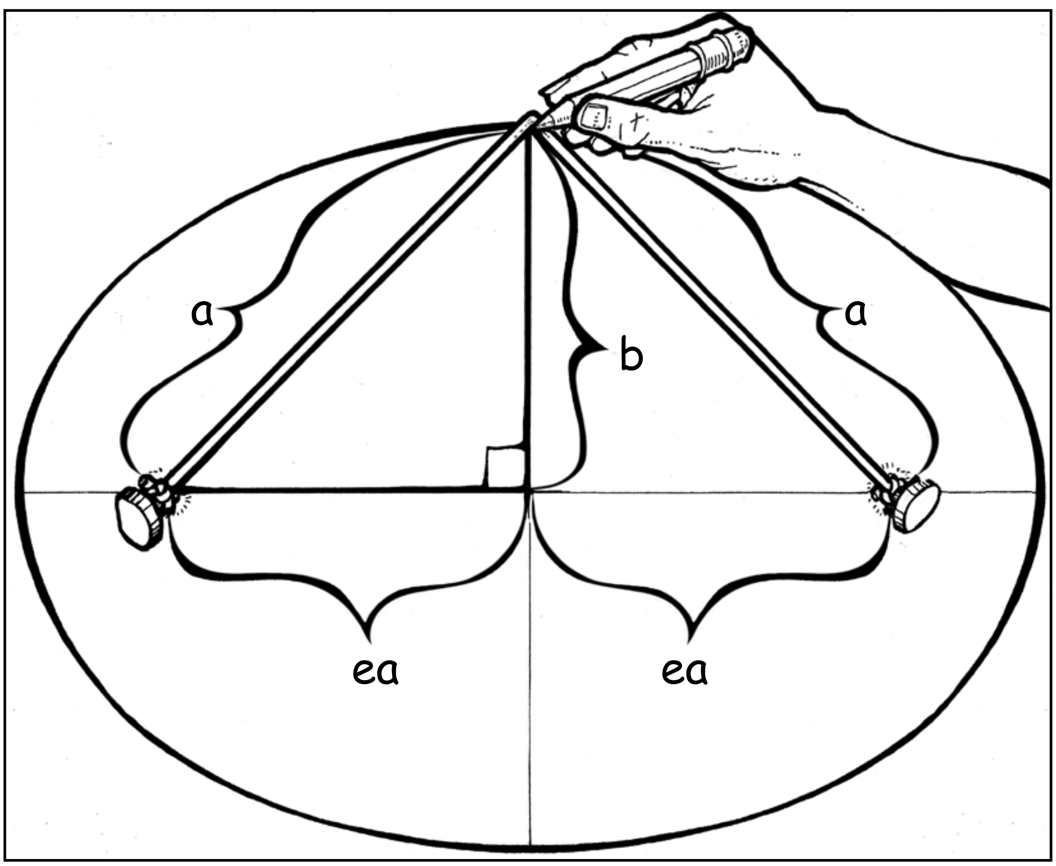
π is a number a little more than 3, about 3.14. It's spelled "pi" and pronounced "pie", like delicious apple pie.

The area of a circle is $\pi r \times r$ which is πr^2.

A circle of radius 10 units has area of about 3.14×10^2 square units, which is 314 units^2.



Snip off the shorter string segment and put it on the other side and you'll see
the string length is 2a, the length of the ellipse's major axis.



b and ea are legs of a right triangle with hypotenuse a.

From the Pythagorean Theorem, page 21:

$(ea)^2 + b^2 = a^2$

$b^2 = a^2 - (ea)^2$

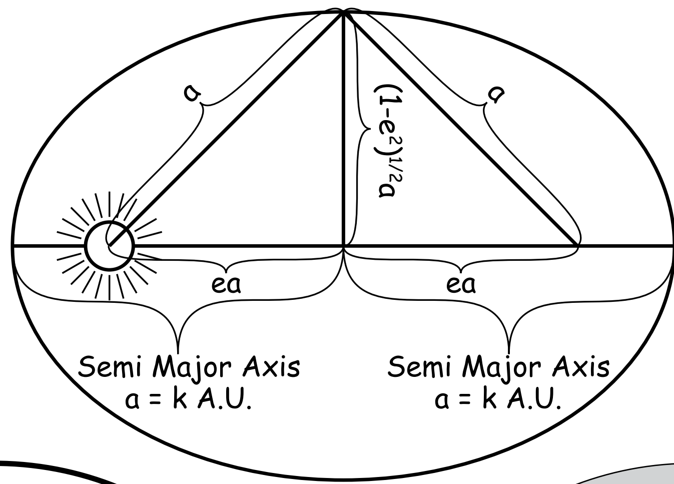
$b^2 = (1 - e^2)a^2$

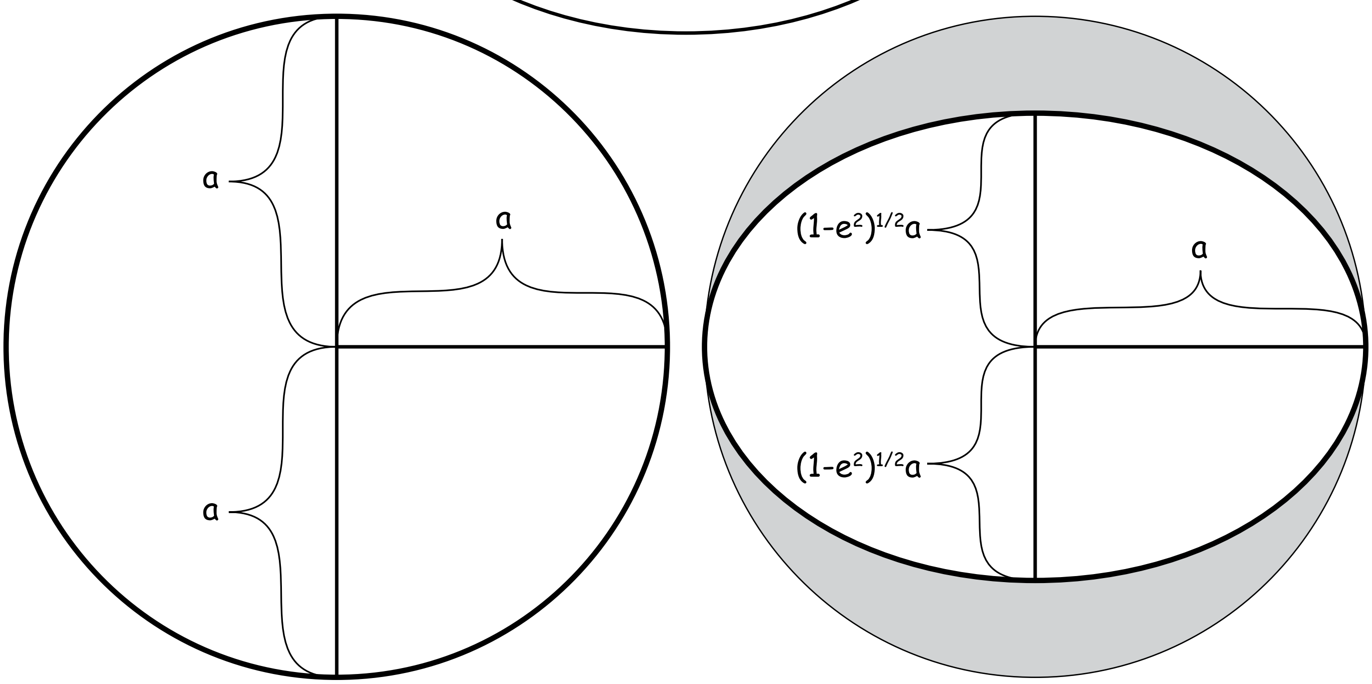
$b = (1 - e^2)^{1/2}a$

$$b = (1 - e^2)^{1/2}a$$

The ellipse with semi-major axis a and eccentricity e
is the circle with radius a vertically scaled by $(1-e^2)^{1/2}$

Area of this ellipse $= (1-e^2)^{1/2} \pi a^2$





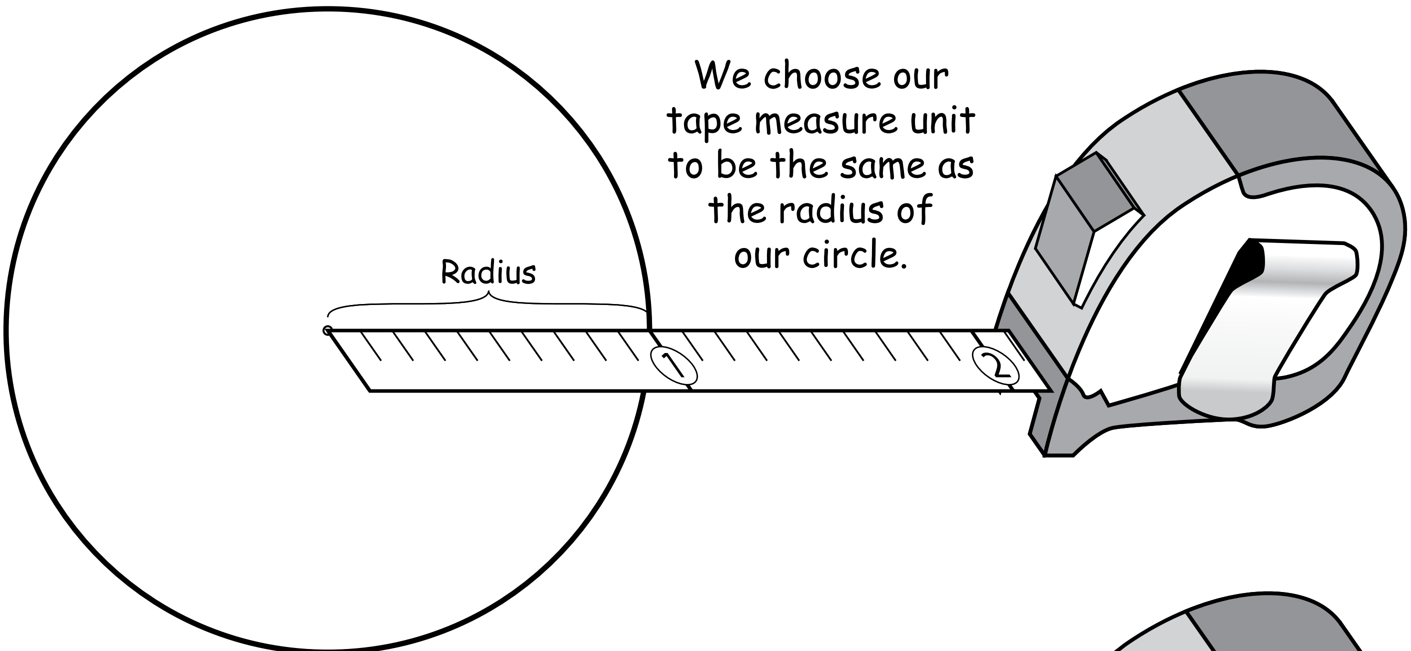
Area $= \pi a^2$

Area ellipse $= (1-e^2)^{1/2} \pi a^2$

$|\mathbf{r} \times \mathbf{v}| =$
Twice area ellipse / orbital period
$= 2\ (1-e^2)^{1/2} \pi\ a^2\ /\ T$
$= 2\ (1-e^2)^{1/2} \pi\ (k\ \text{A.U.})^2\ /\ (k^{3/2}\ \text{years})$
$= 2\ (1-e^2)^{1/2} \pi\ k^{1/2}\ \text{A.U.}^2\ /\ \text{year}$

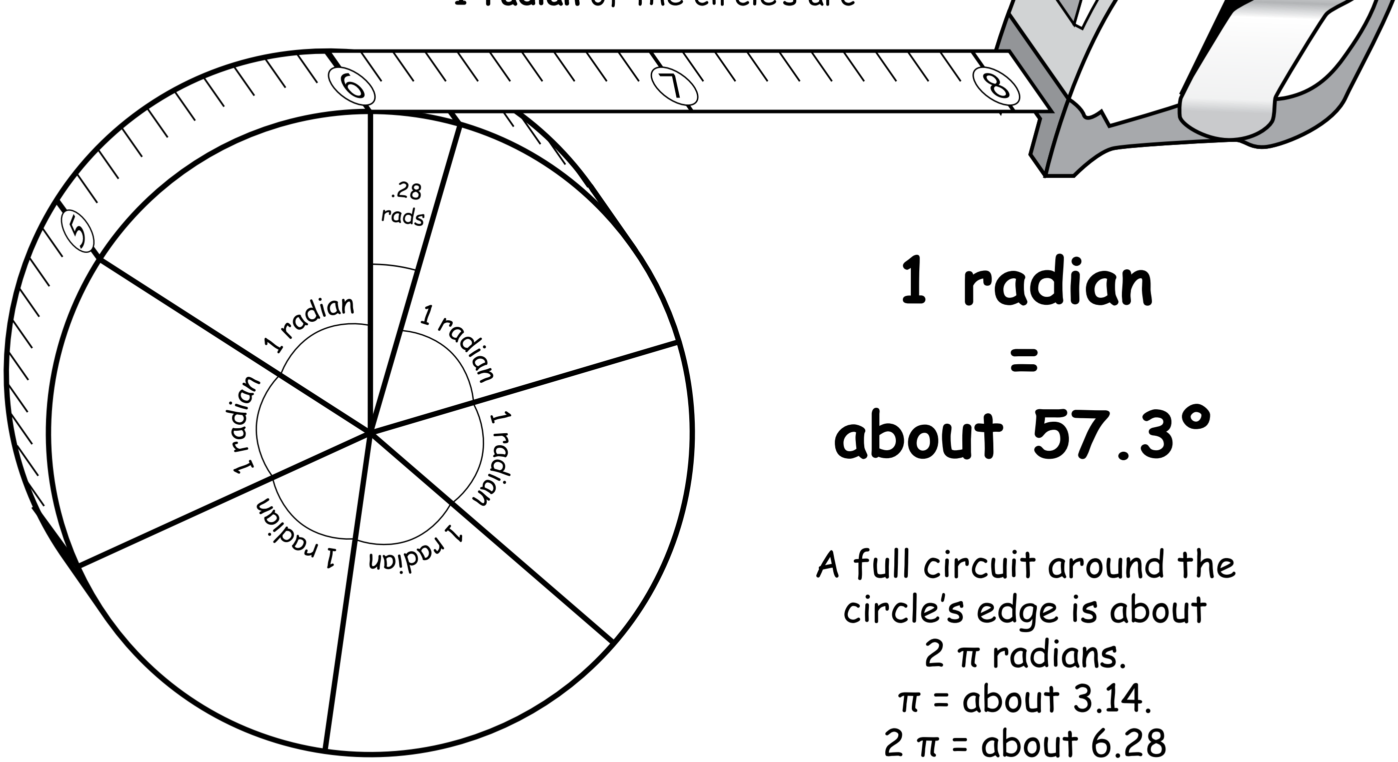
An ellipse can be thought of as a circle shrunk along one of it's diameters.
Thus the area of the ellipse is the area of the circle shrunk by the same factor.
Specific angular momentum $|\mathbf{r} \times \mathbf{v}|$ is twice area ellipse over orbital period.

RADIANS



We choose our tape measure unit to be the same as the radius of our circle.

Wrap the tape around the circle's circumference. Each unit on the tape covers **1 radian** of the circle's arc

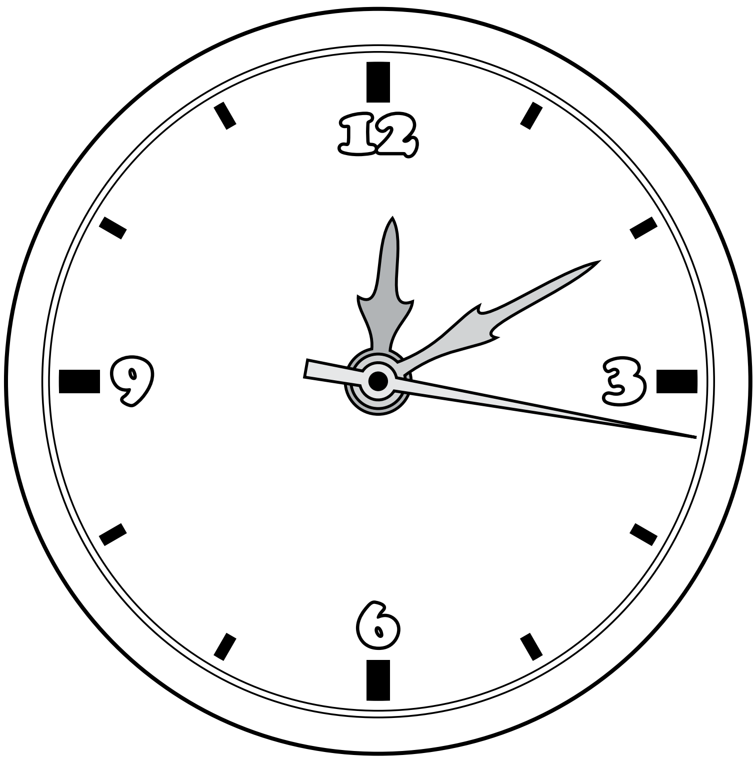


1 radian = about 57.3°

A full circuit around the circle's edge is about 2π radians.
π = about 3.14.
2π = about 6.28

ω

ω is the Greek lower case letter omega.



The symbol ω is often used to denote **angular velocity** in radians covered over a period of time.

A full circuit is 2 π radians

Examples:

The second hand on a clock has
ω = 2 π radians / minute

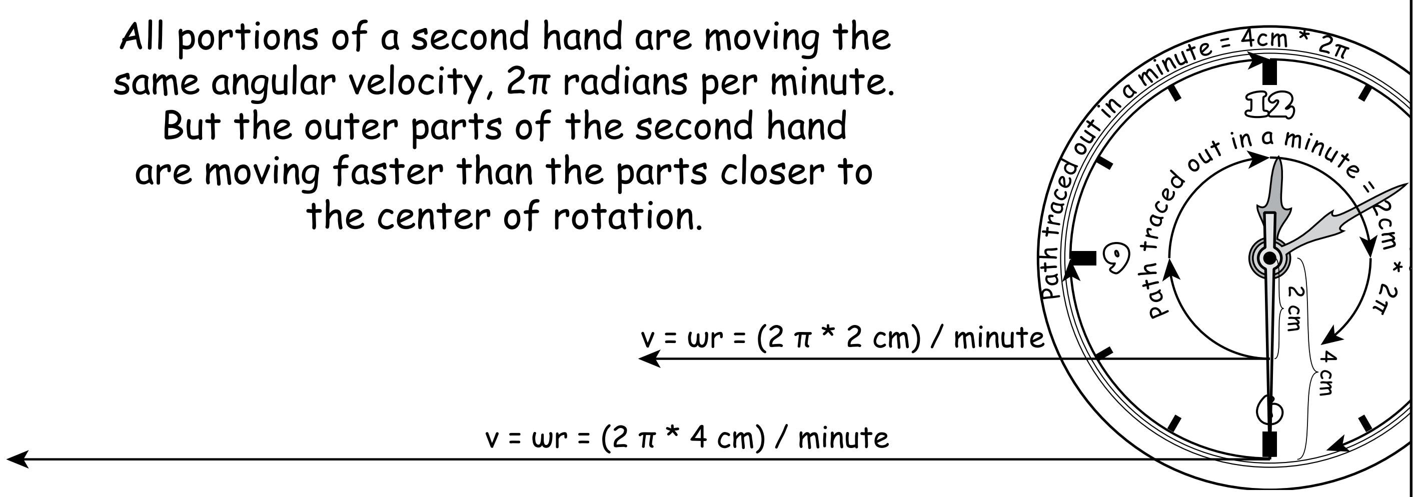
The minute hand on a clock has
ω = 2 π radians / hour

The hour hand on a clock has
ω = 2 π radians / 12 hours

Speed is angular velocity in radians times r where r is distance from center of rotation.

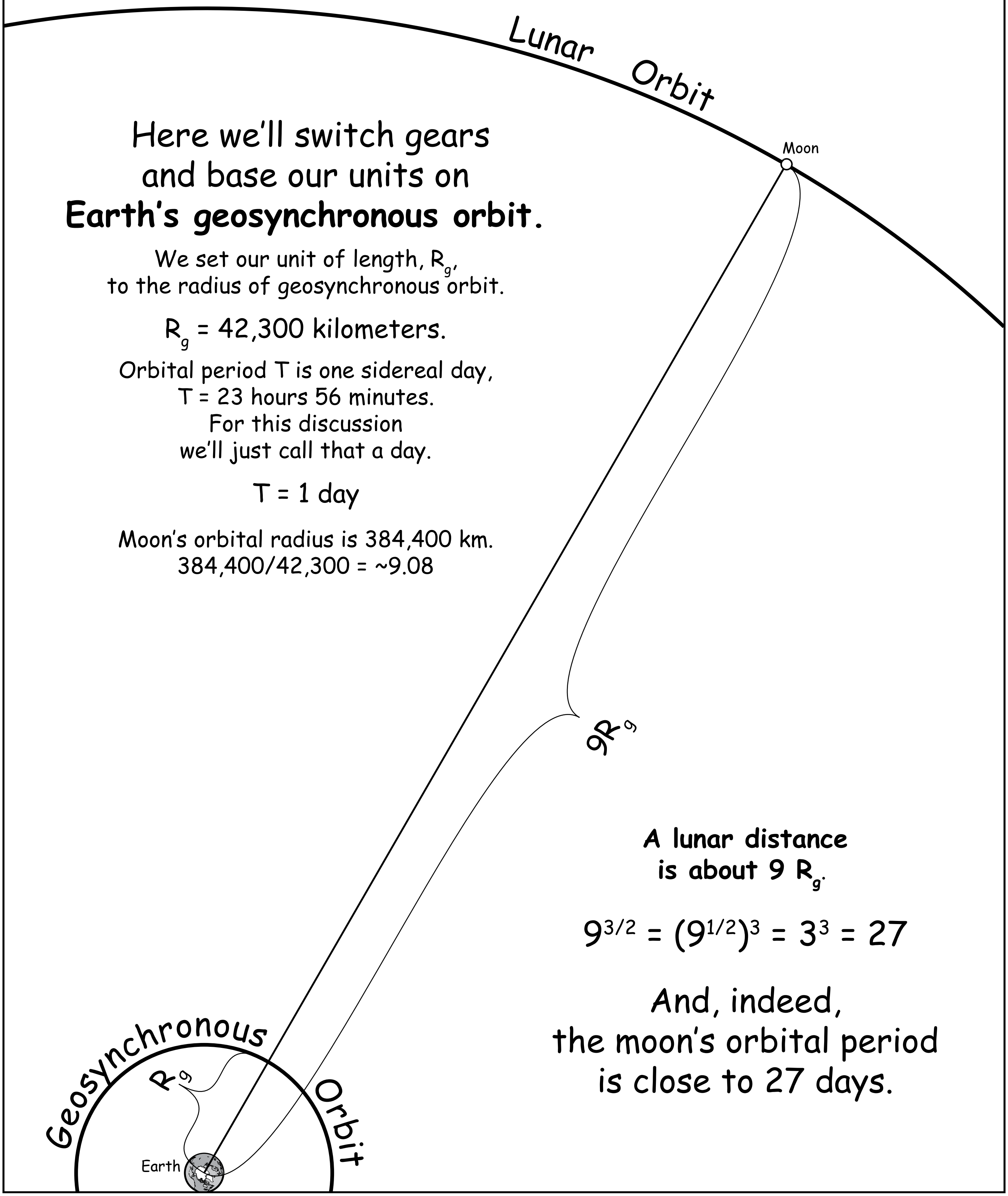
$$v = \omega r$$

All portions of a second hand are moving the same angular velocity, 2π radians per minute. But the outer parts of the second hand are moving faster than the parts closer to the center of rotation.



We've been using canonical units based on earth's orbit around the sun.

But we can also choose canonical units based on any circular orbit around any body. Kepler's Third Law still applies.



Here we'll switch gears and base our units on **Earth's geosynchronous orbit.**

We set our unit of length, R_g, to the radius of geosynchronous orbit.

R_g = 42,300 kilometers.

Orbital period T is one sidereal day, T = 23 hours 56 minutes. For this discussion we'll just call that a day.

T = 1 day

Moon's orbital radius is 384,400 km. 384,400/42,300 = ~9.08

A lunar distance is about 9 R_g.

$$9^{3/2} = (9^{1/2})^3 = 3^3 = 27$$

And, indeed, the moon's orbital period is close to 27 days.

Gravity Gradient Stabilized Vertical Tethers

A.K.A. Sarmount Sky Hooks

The upper parts of such tethers feel an upward net pull.

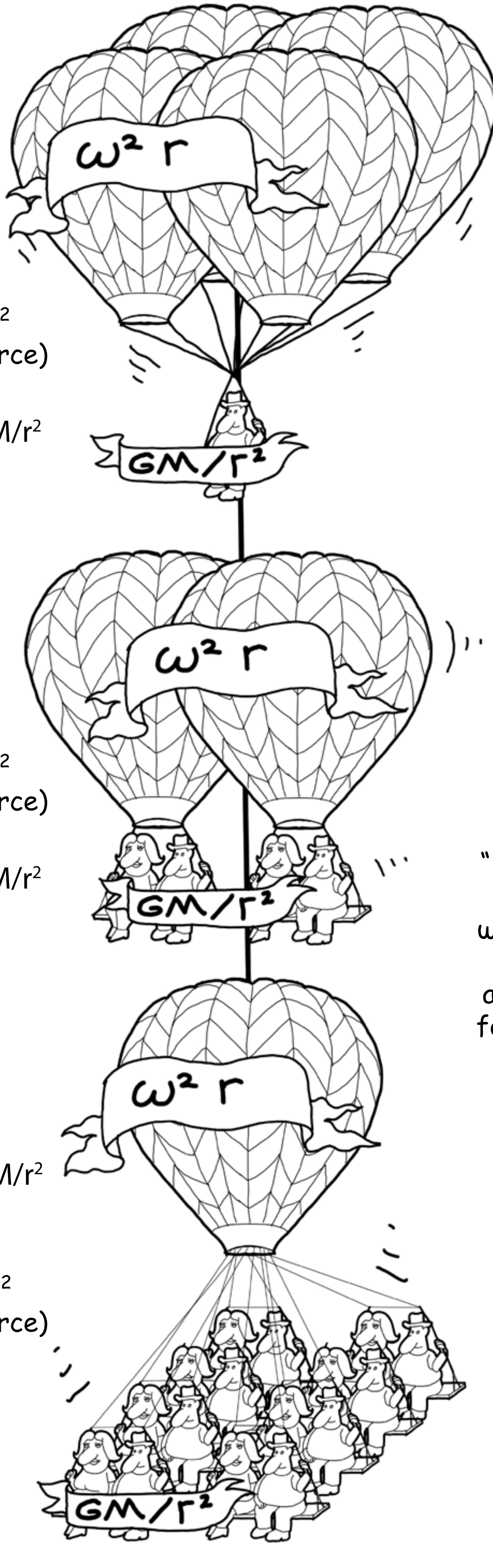
The lower parts feel a net downward acceleration.

Tidal forces keep such tethers aligned to the local vertical.

Upward ωr^2 (centrifugal force) **exceeds** downward GM/r^2 (gravity)

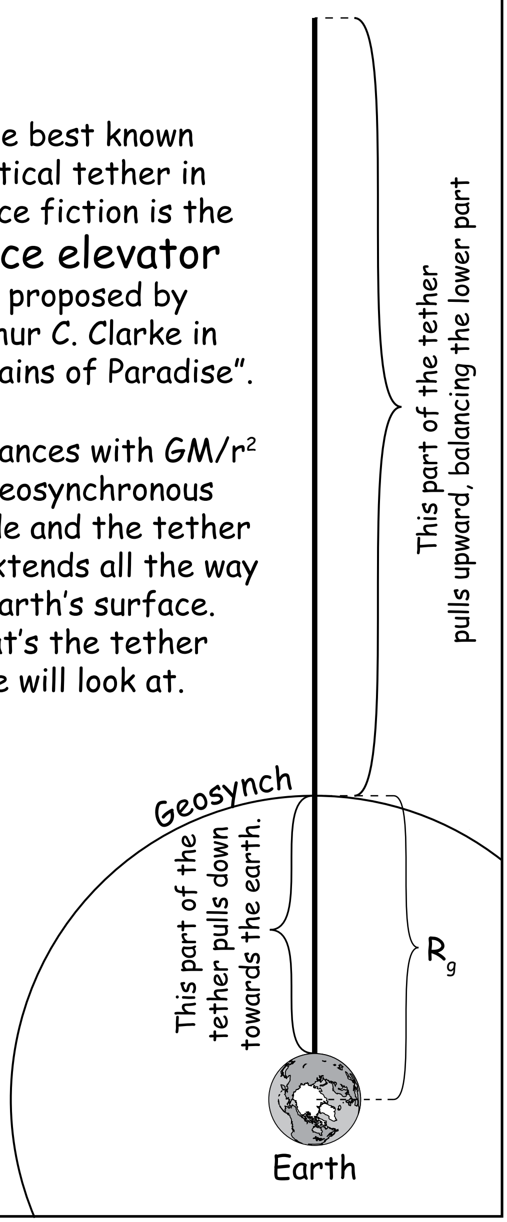
Upward ωr^2 (centrifugal force) **balances** downward GM/r^2 (gravity)

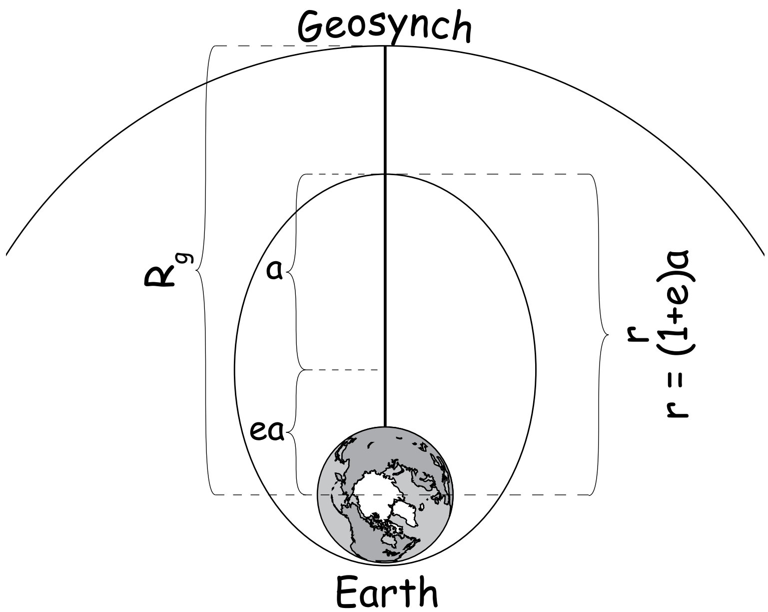
Downward GM/r^2 (gravity) **Exceeds** upward ωr^2 (centrifugal force)



The best known vertical tether in science fiction is the **space elevator** as proposed by Arthur C. Clarke in "Fountains of Paradise".

ωr^2 balances with GM/r^2 at geosynchronous atlitude and the tether foot extends all the way to earth's surface. That's the tether we will look at.



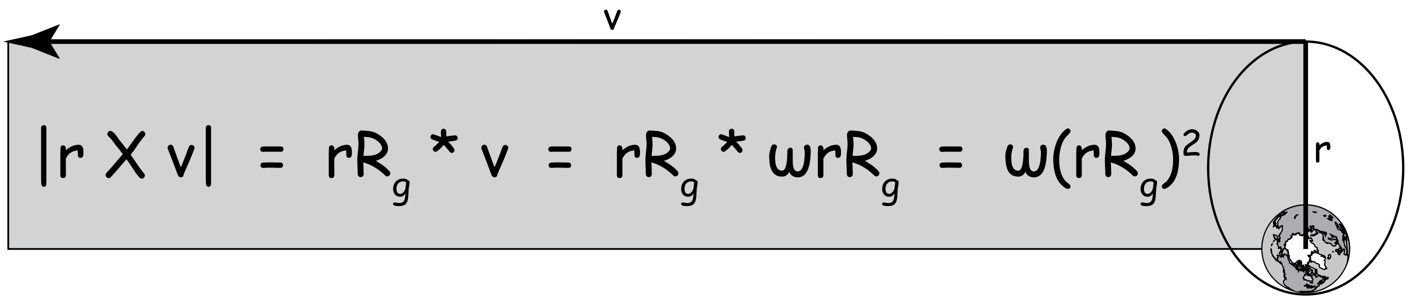


Take a point on the beanstalk.
Call the distance from this point
to earth's center $r R_g$.

Note we're using
R_g as our unit of length.

Release a payload from this point and
it will fall into an elliptical orbit with
earth's center at a focus and
r is the apogee of this ellipse.

$$r R_g = (1+e)a$$



$$|r \times v| = rR_g * v = rR_g * \omega r R_g = \omega(rR_g)^2$$

Every point on the elevator is moving at the same angular velocity, 2 π radians/day.

An alert reader might say "Hey! That rectangle's area
is a lot more than twice the are of the ellipse!"

That's because we are using a day as our time unit.
ωr would be shorter if we used T, the orbital period of this ellipse, as our time unit, .

|r X v| = twice ellipse area/ellipse's orbital period

$$\omega(rR_g)^2 = (1-e^2)^{1/2} * 2 \pi a^2 /T$$

Recall $a = k R_g$.

$$\omega(rR_g)^2 = (1-e^2)^{1/2} * 2 \pi (kR_g)^2 /(k^{3/2} \text{ days})$$

$$2 \pi/\text{day} * (rR_g)^2 = (1-e^2)^{1/2} * 2 \pi k^{1/2} * R_g^2 /\text{day}$$

$$(rR_g)^2 = (1-e^2)^{1/2} * k^{1/2} * R_g^2$$

$$r^2 = (k(1-e^2))^{1/2}$$

Now $rR_g = (1+e)a$ which $= (1+e)kR_g$ so $k = r/(1+e)$

$$r^2 = (r(1-e^2)/(1+e))^{1/2}$$

$$r^4 = r(1-e^2)/(1+e)$$

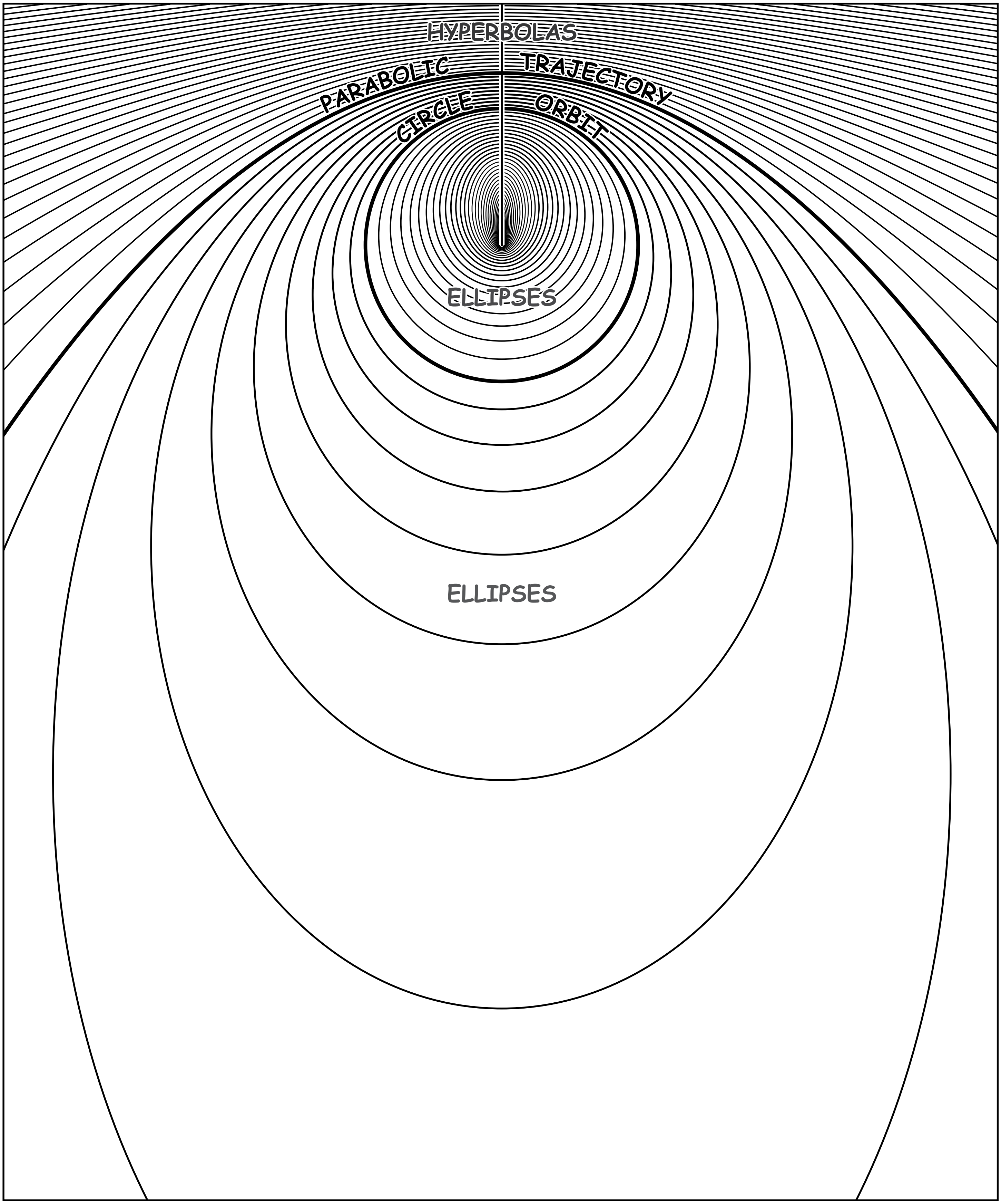
$$r^3 = 1-e$$

$$e = 1- r^3$$

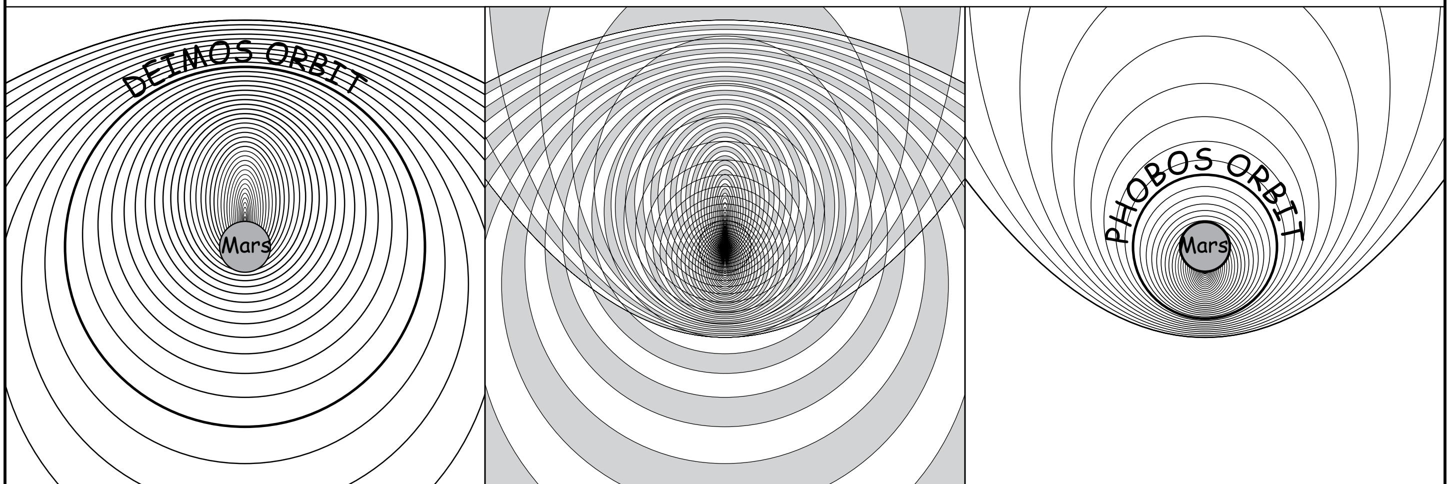
If r > 1, payload is released at perigee and we can use similar methods to find $e = r^3-1$.
In general

$$e = |r^3 - 1|$$

So we know the eccentricity of the conic payload follows when released from the elevator. This plus the fact that release point is at either periapsis or apoapsis of the orbit allows us to draw a family of conics associated with the elevator



Zero Relative Velocity Transfer Orbit



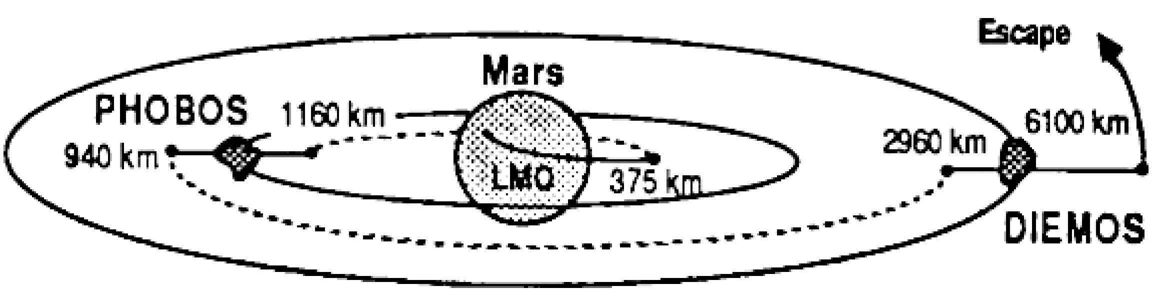
Anchor a vertical elevator on the Martian moon Deimos.
Between Deimos circular orbit and Mars' center
there are ellipses of every eccentricty between 0 and 1.

Anchor an elevator at the Martian moon Phobos.
Between Phobos circular orbit and the parabola
there are also ellipses of every eccentricity between 0 and 1.

Do the Phobos and Deimos elevators share an ellipse?
Overlapping the two families of conics, the moiré pattern seems to indicate a shared ellipse.

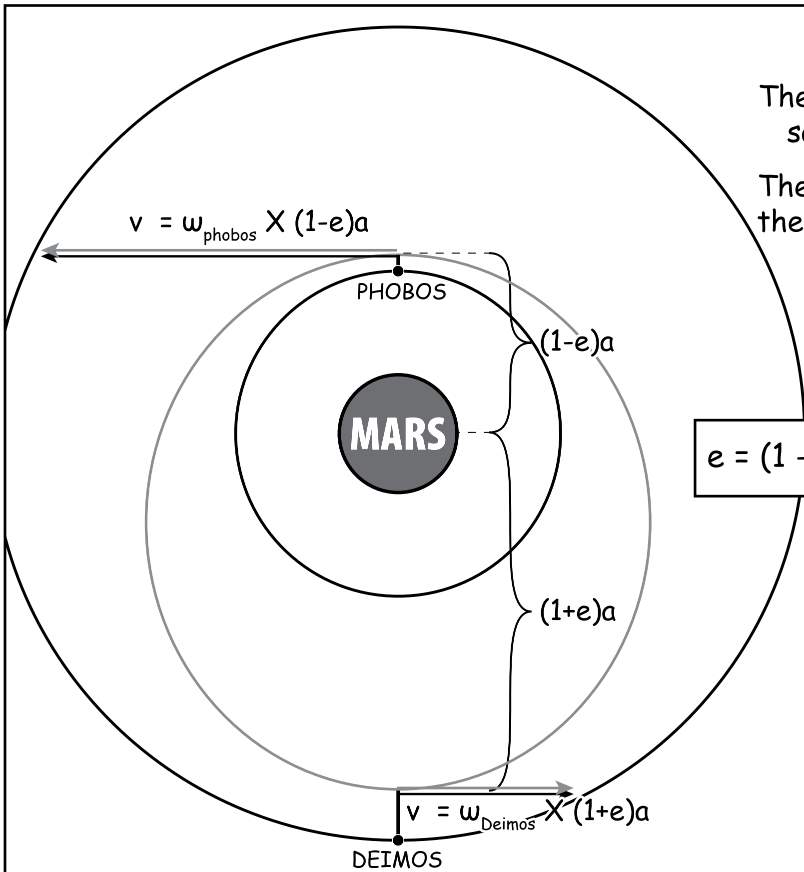
At periapsis a payload traveling along this elliptical orbit would have the same relative velocity as the rendezvous point on a Phobos elevator. At apoapsis the payload would have the same relative velocity as the rendezvous point on a Deimos tether.

Using this **Zero Relative Velocity Transfer Orbit** the two moons **could exchange payloads using virtually zero reaction mass.**



Paul Penzo, a JPL engineer, talked about this possible path between Deimos and Phobos elevators back in 1984. Above is Penzo's illustration from that paper.

I believe ZRVTO is a term coined by Marshall Eubanks who is also an advocate of PAMSE -- Phobos Anchored Mars Space Elevator.



The top of the Phobos tether is moving the same angular velocity as Phobos, ω_{phobos}.

The bottom of the Deimos tether is moving the same angular velocity as Deimos, ω_{Deimos}.

Specific angmom = $v_{periaerion} \times r_{periaerion}$

Specific angmom = $v_{apoiaerion} \times r_{apoiaerion}$

$v_{periaerion} \times r_{periaerion} = v_{apoiaerion} \times r_{apoiaerion}$

$\omega_{Phobos} \times ((1-e)a)^2 = \omega_{Deimos} \times ((1+e)a)^2$

$$e = (1 - (\omega_{Deimos}/\omega_{Phobos})^{1/2})/(1 + (\omega_{Deimos}/\omega_{Phobos})^{1/2})$$

Specific angmom =

$\omega_{Phobos} \times r^2 = (a(1-e^2)\mu)^{1/2}$

At periapsis r is (1-e)a.

So a = r/(1-e). Substituting:

$\omega_{Phobos} \times r^2 = (r(1+e)\mu)^{1/2}$

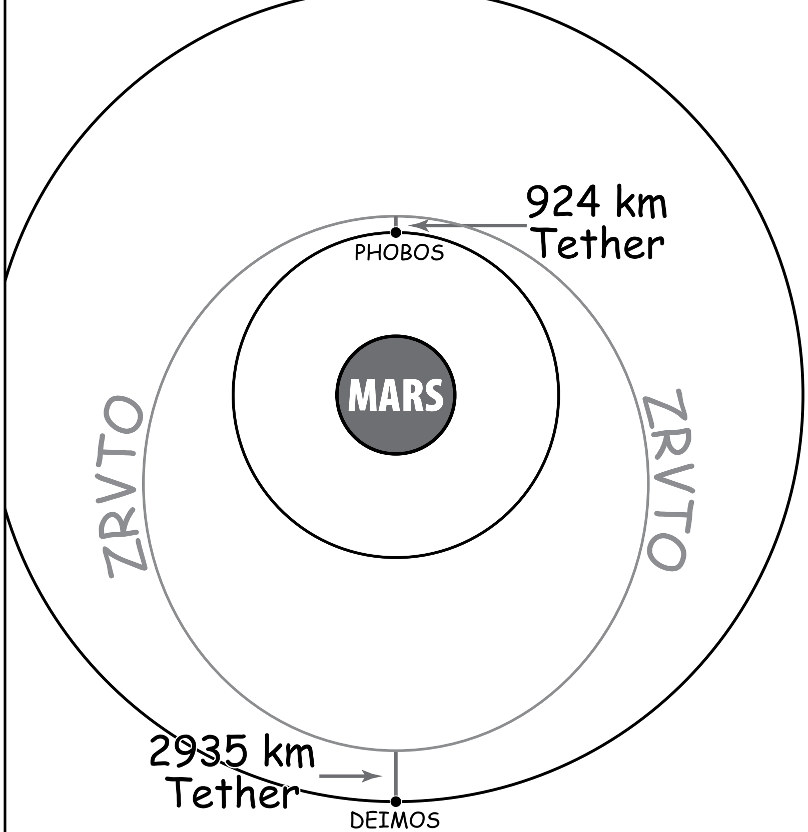
$r^4 = r(1+e)\mu/\omega_{Phobos}{}^2$

$r = ((1+e)\mu/\omega_{Phobos}{}^2)^{1/3}$

$$r_{periaerion} = (1 + e)^{1/3} r_{Phobos}$$

Similarly:

$$r_{apoaerion} = (1 - e)^{1/3} r_{Deimos}$$



Angular velocities as well as orbital radii of Phobos and Deimos are easily found on Wikipedia.

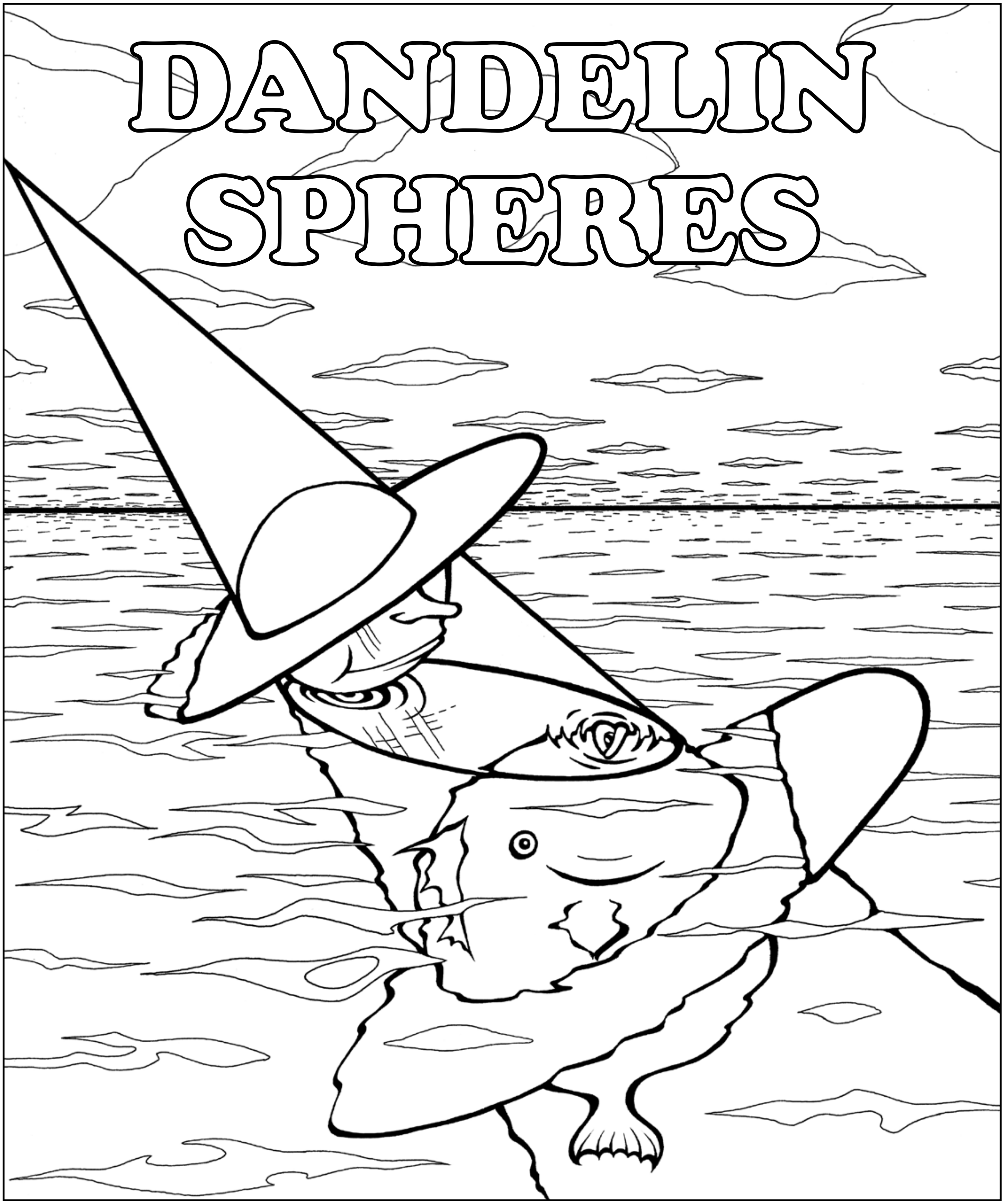
Plugging these into the above equations we find an ~1000 km tether ascending from Phobos and a ~3000 km tether descending from Deimos is sufficient for a ZRVTO route between the two moons.

Not just Phobos & Deimos

This technique can be used for any pair of tide-locked moons in nearly circular, coplanar orbits.

Anchor moons could be man made. A series of orbital tethers would be shorter and endure less stress than a full blown space elevator to a planet's surface.

DANDELIN SPHERES



A floating ball head is wearing a dunce cap/mosquito net. Where the ocean meets the mosquito net is an ellipse. Where the ball head touches the water is a focus. Where the fish kisses the air is a focus. The ball head's hat brim is a directrix plane as is the fish's belt plane. Where the directrix planes meet the ocean surface are two lines called directrix lines.

Each radius of a circle has length r.

A line tangent to the circle is at right angles to the radius it touches.

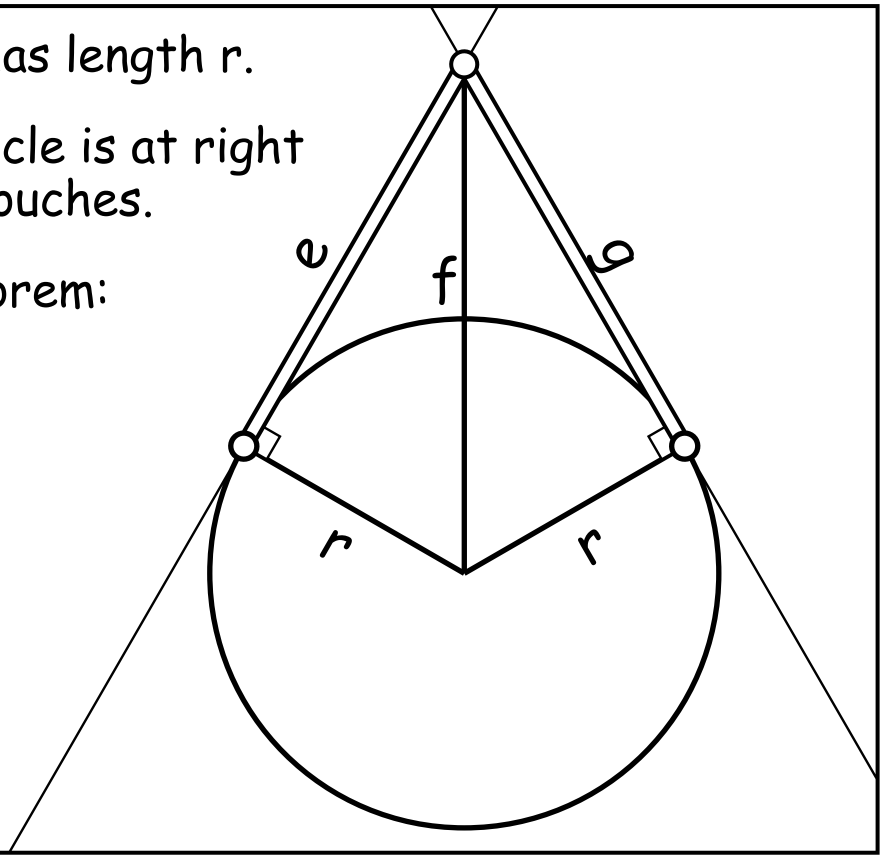
by the Pythagorean theorem:

$e^2 + r^2 = f^2 \quad e^2 = f^2 - r^2$

$g^2 + r^2 = f^2 \quad g^2 = f^2 - r^2$

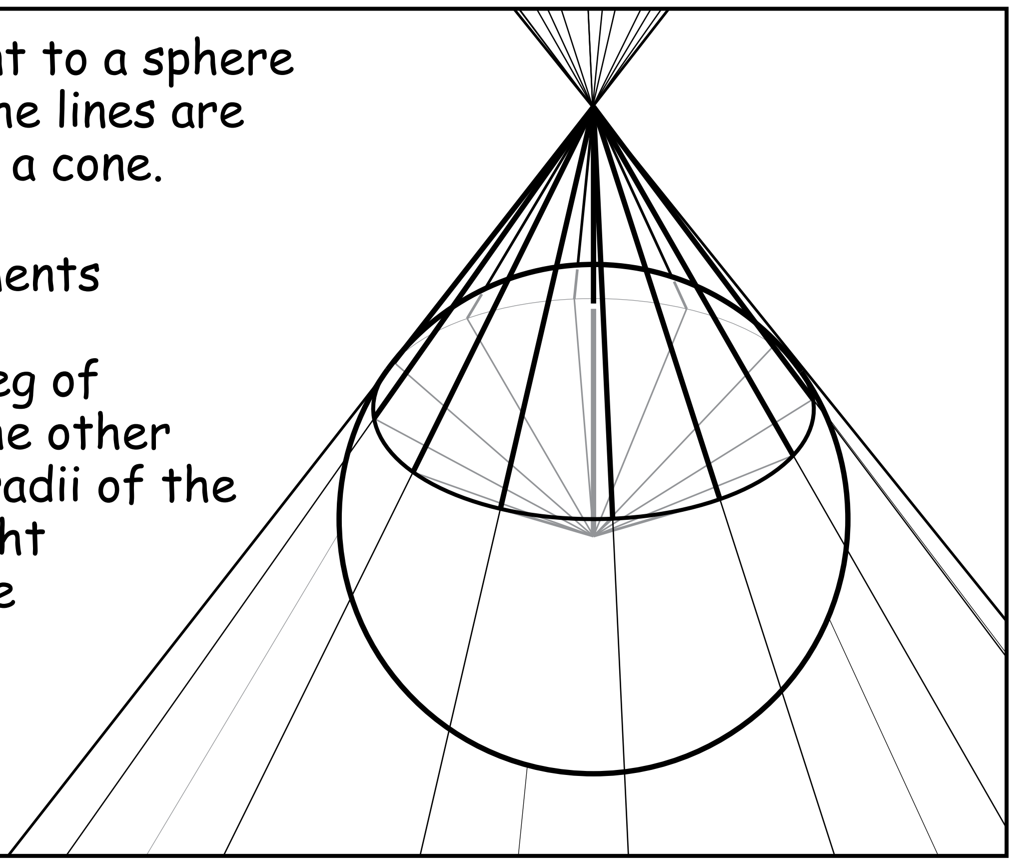
e = g

Two such line segments on tangent lines whose end points meet are equal.

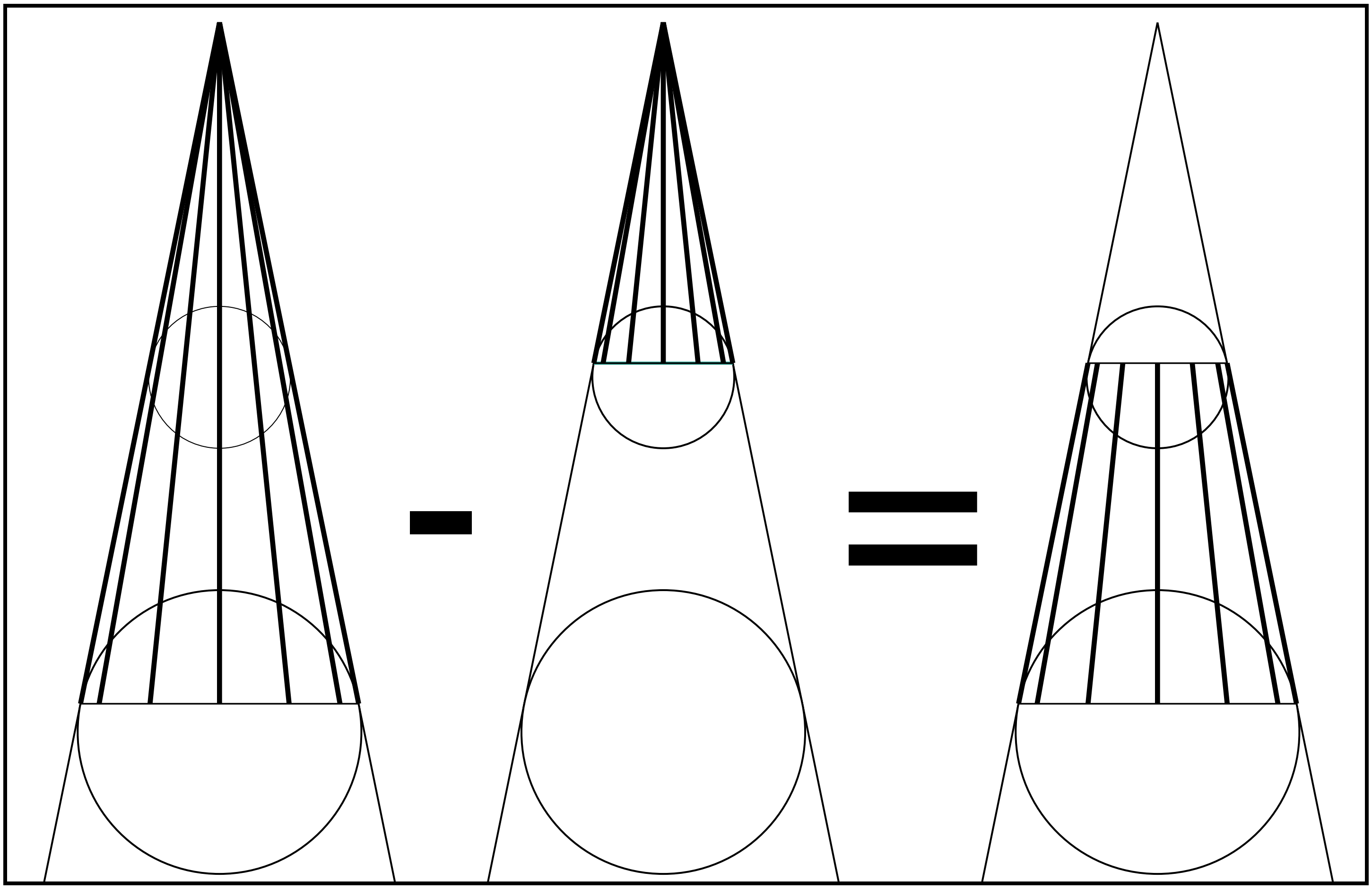


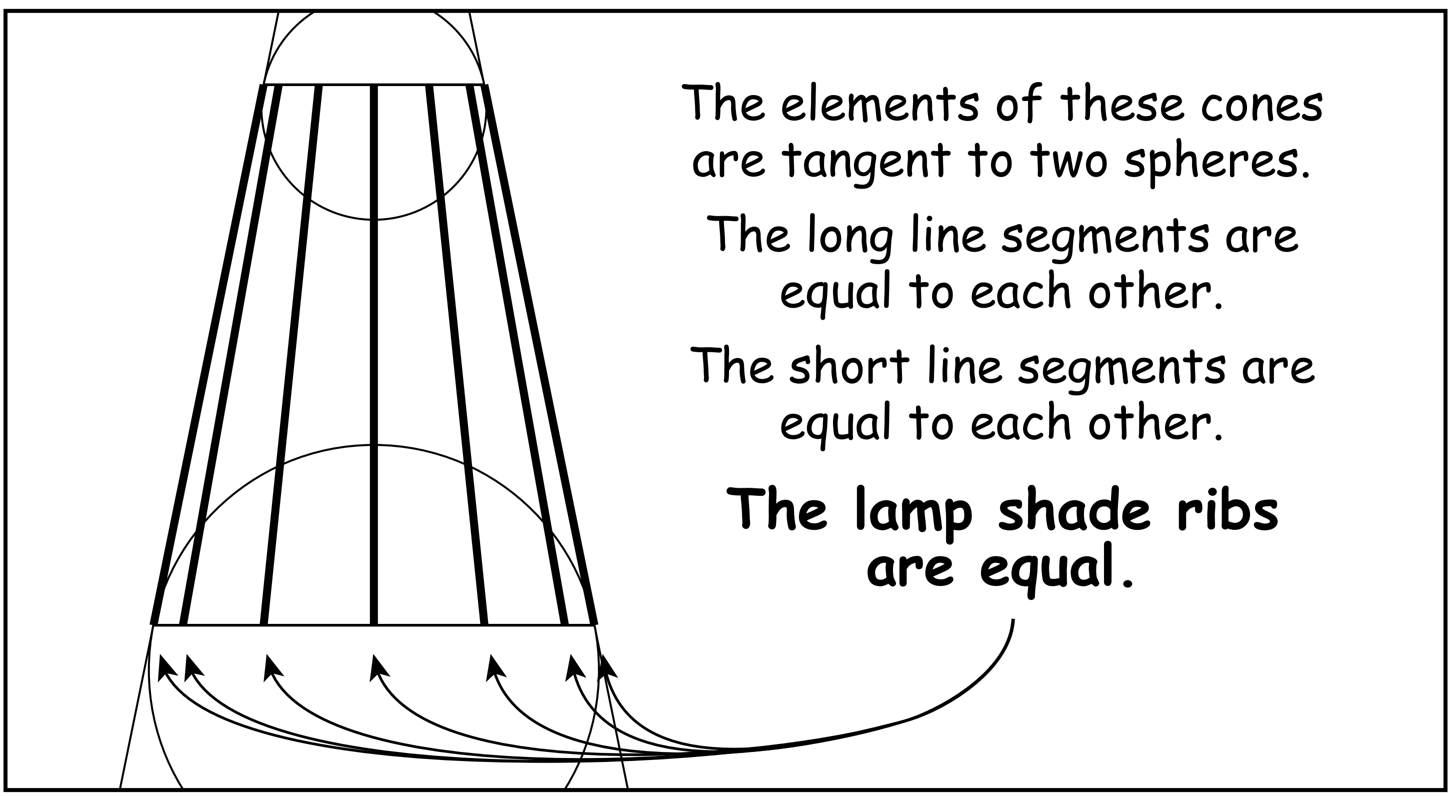
These lines tangent to a sphere meet at a point. The lines are called elements of a cone.

The bold line segments are all equal. Each line segment is a leg of a right triangle, the other leg being a circle radii of the sphere. All the right triangles share the same hypotenuse.

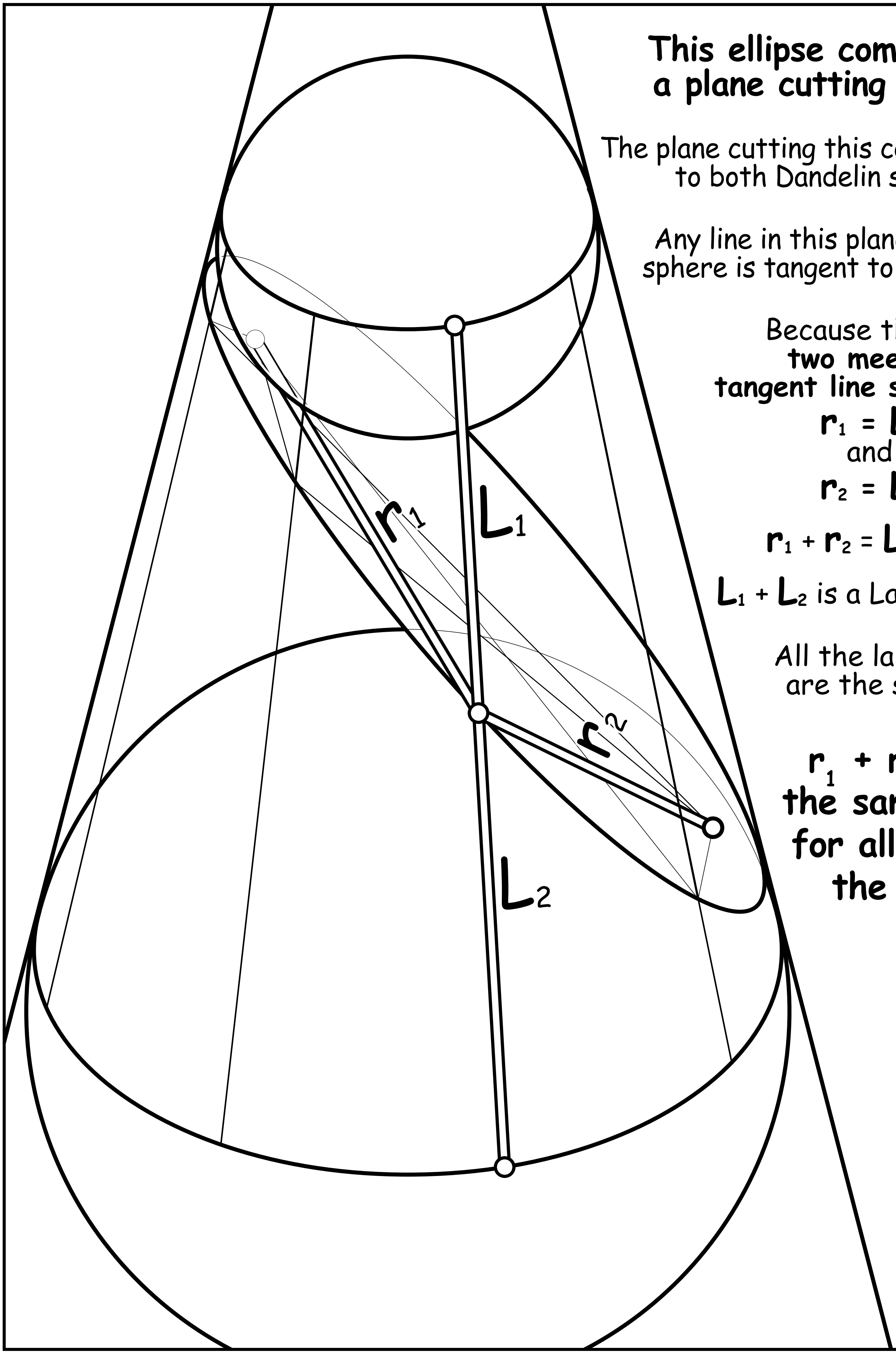


The equality of line segments whose ends meet, that lie on lines tangent to the sphere and having an end lieing on the sphere, is a tool in use of **Dandelin Spheres**.





If a = b and c = d, then a - c = b - d.
Each rib of the above lamp shade ia
a line segment equal to each other rib.



This ellipse comes from a plane cutting a cone.

The plane cutting this cone is tangent to both Dandelin spheres.

Any line in this plane touching a sphere is tangent to that sphere.

Because they're **two meeting tangent line segments,**

$$r_1 = L_1$$

and

$$r_2 = L_2$$

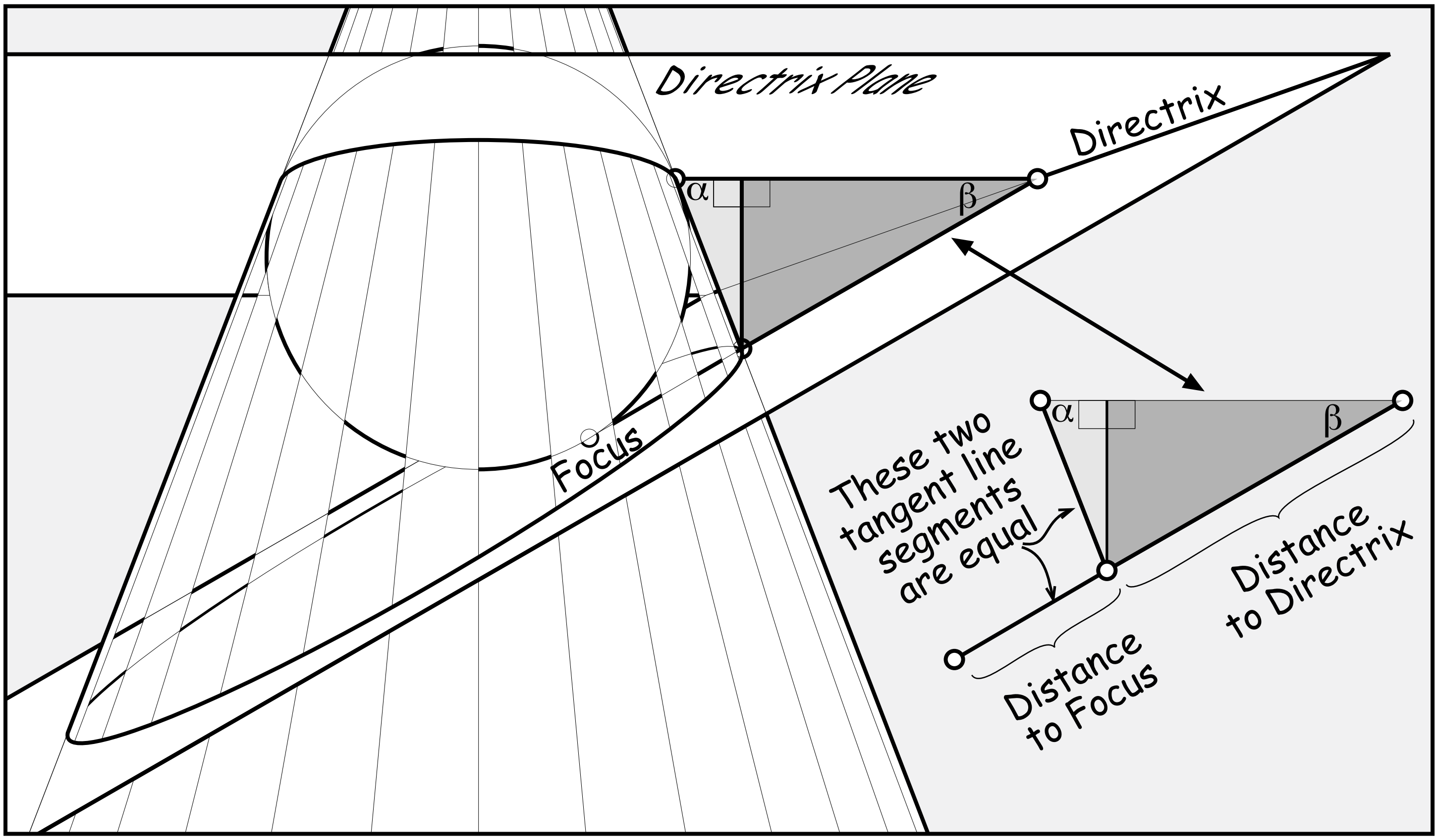
$$r_1 + r_2 = L_1 + L_2$$

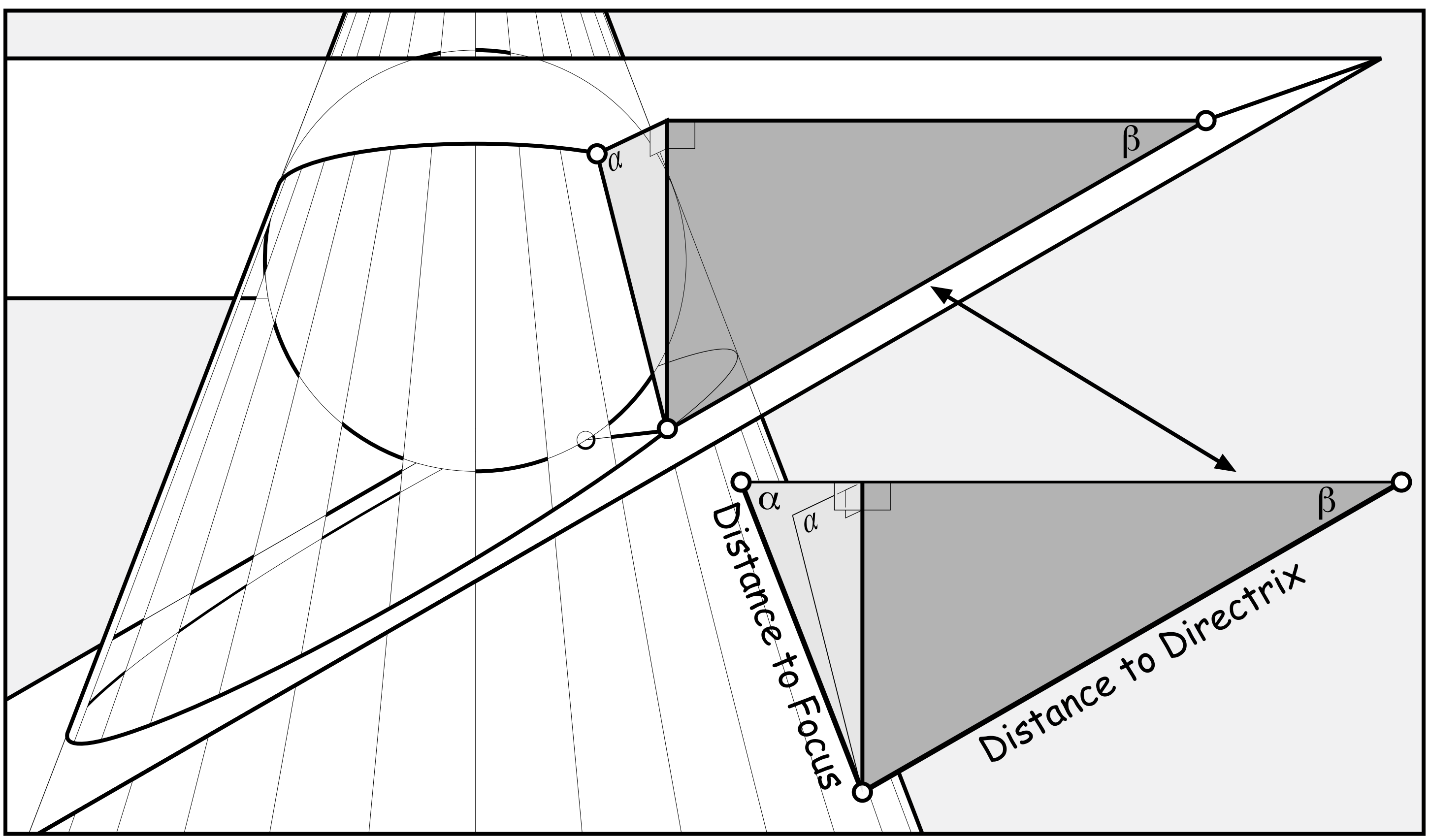
$L_1 + L_2$ is a Lampshade rib.

All the lampshade ribs are the same length.

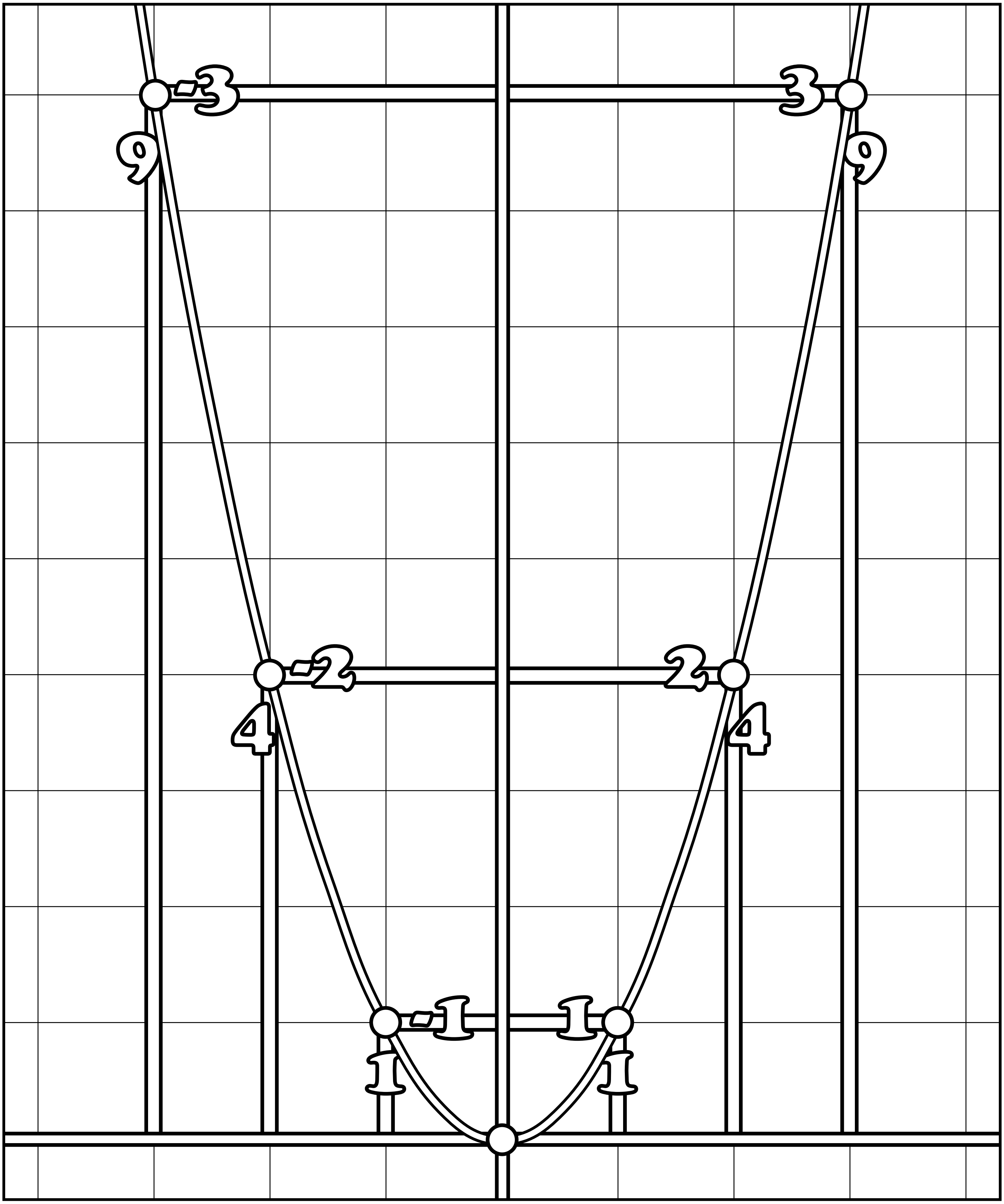
$r_1 + r_2$ sum to the same number for all points on the ellipse.

Dandelin spheres show that two descriptions of the ellipse do indeed describe the same thing.

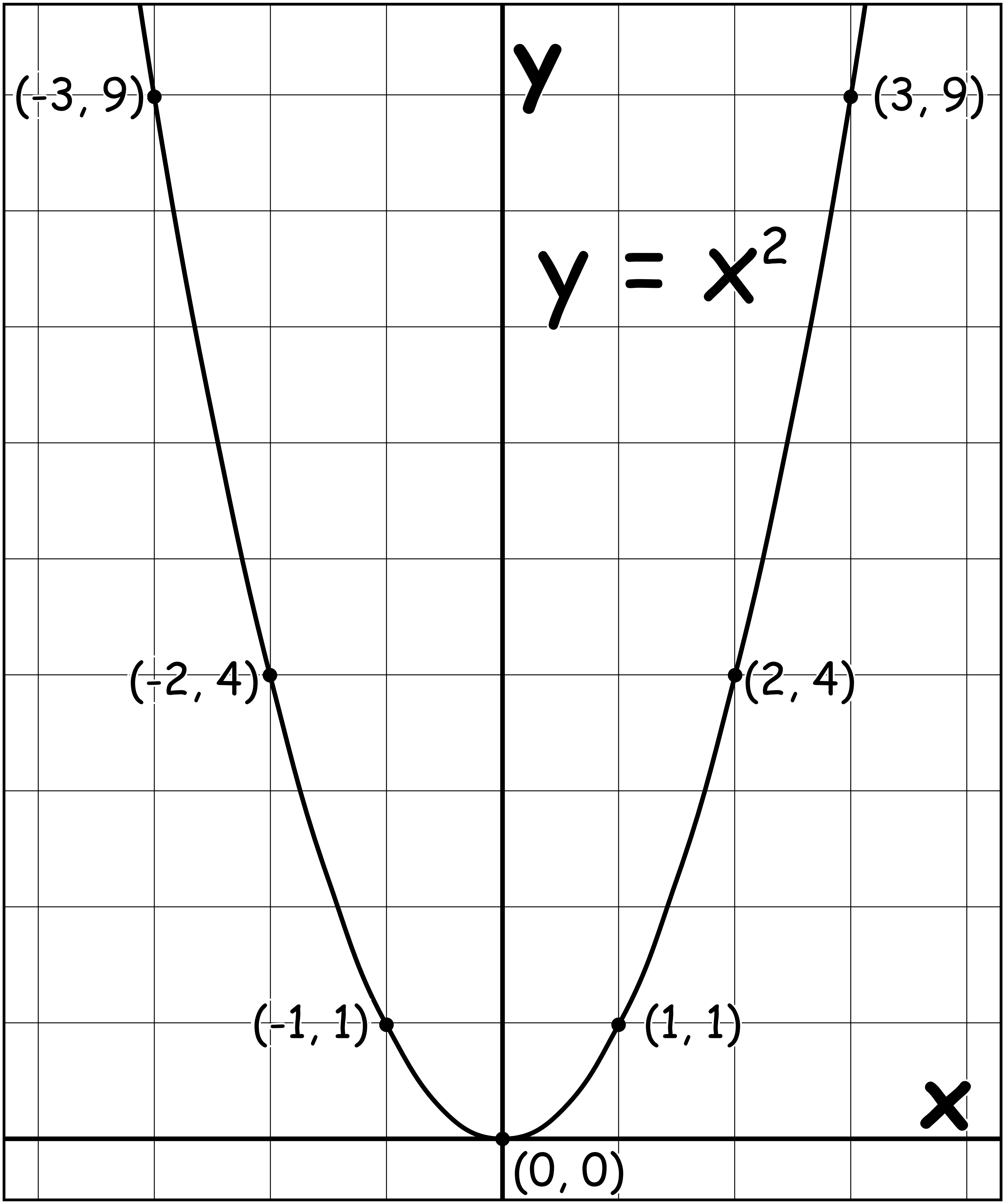




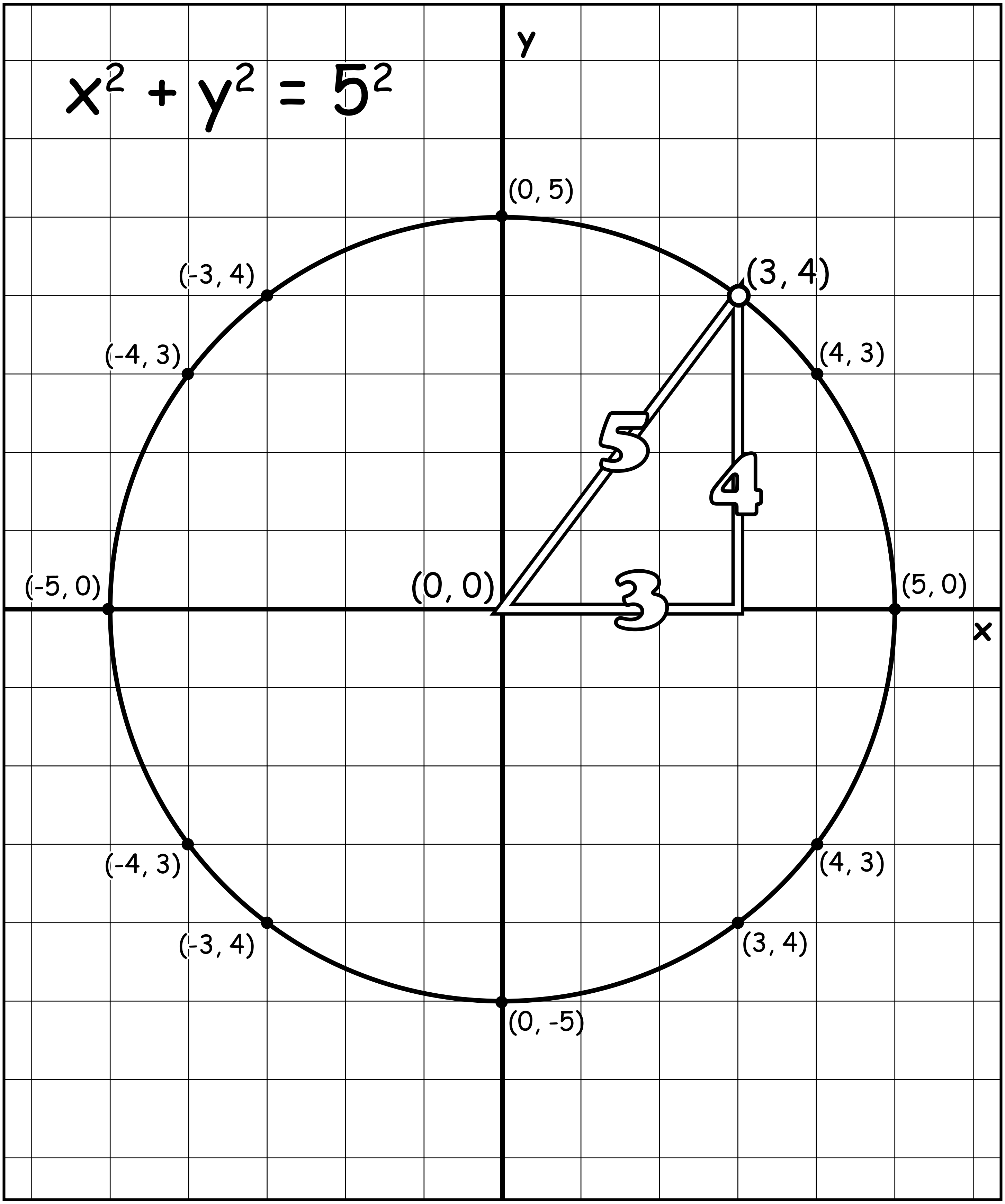
Drop a line segment straight down from the directrix plane to a point on the ellipse.
The cone element line segment to the point is the same length as the point's distance to focus. All cone elements meet the directrix plane at angle α. The cutting plane meets the directrix plane at angle β.
The line straight down from the directrix is a fold in a triangle having angles α and β.
All these triangles are similar, having the same proportions.
Since distance to focus and distance to focus are always sides of similar triangles, the ratio of these two lengths remain constant.



Pages 3, 4 & 5 we looked at conics in terms of distance from **a point and a line**.
Pages 10 and 11 we looked at conics in terms of distance from **two points**.
Now we will look at conics in terms of distance from **two lines**.
The vertical line we call the **y axis**, the horizontal line we call the **x axis**.
Above is a picture of a parabola. Can you see a pattern?



Above is the more usual way of showing a parabola on a Cartesian grid.
When (x, y) coordinates are given, the first gives horizontal distance from the y axis, the second coordinate gives vertical distance from the x axis.
Going to the left or going down is given a minus sign.



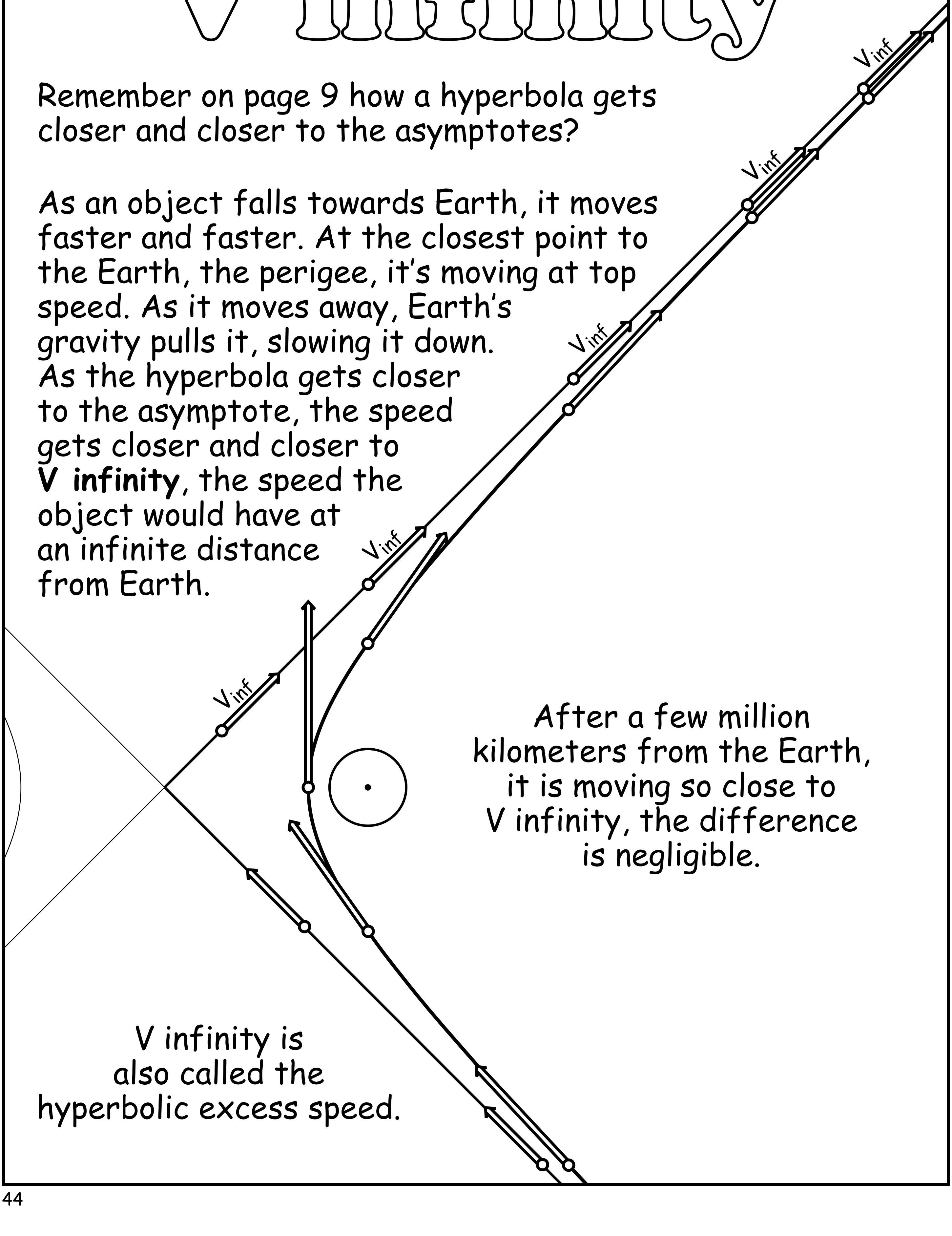
The vertical and horizontal distance can be seen as legs of a right triangle.
Distance from the origin (0, 0) to a point is the hypotenuse of this right triangle.
All these points are 5 units away from the origin.

$x^2 + y^2 = 5^2$ describes a circle with radius 5.

V infinity

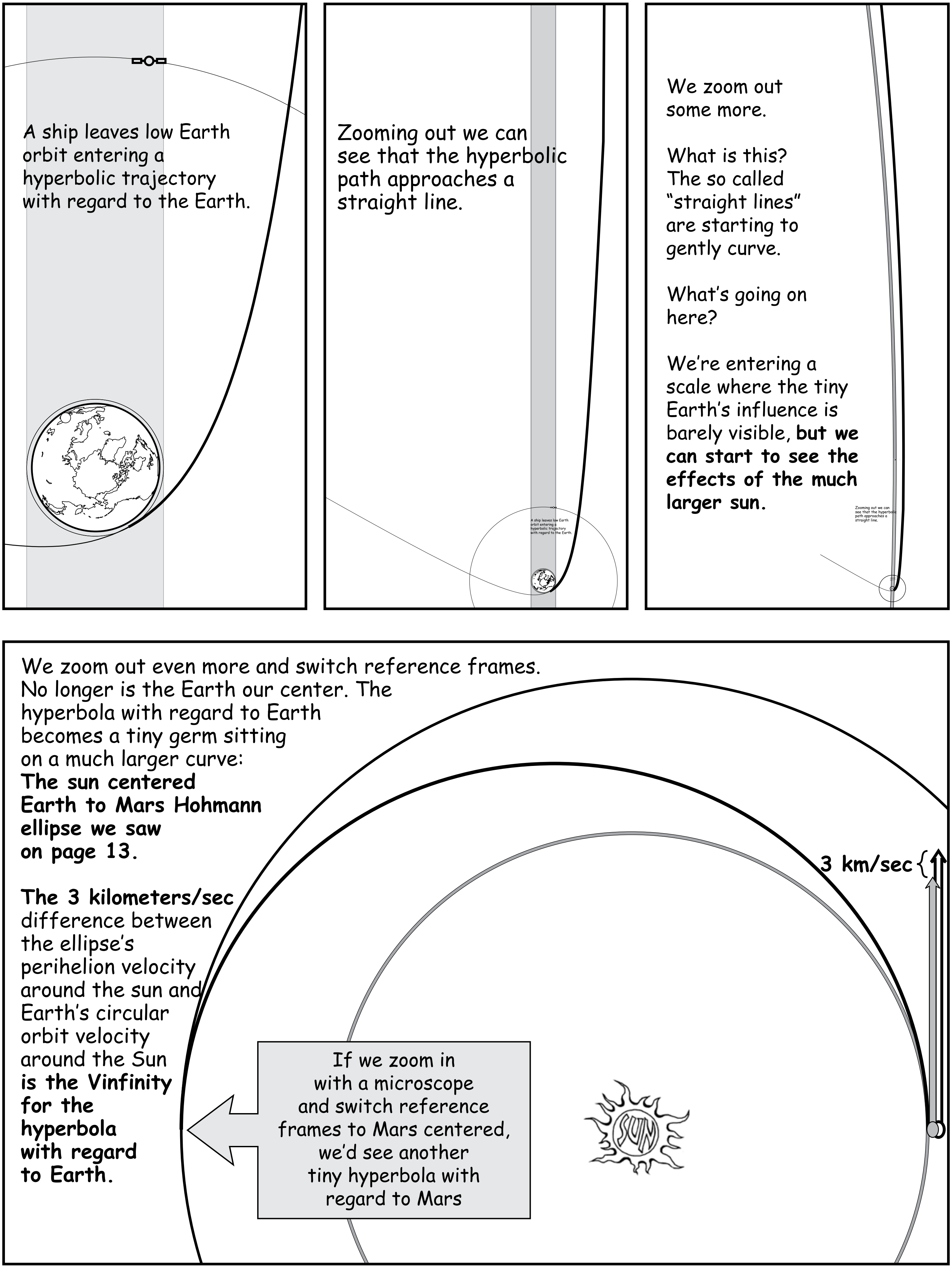
Remember on page 9 how a hyperbola gets closer and closer to the asymptotes?

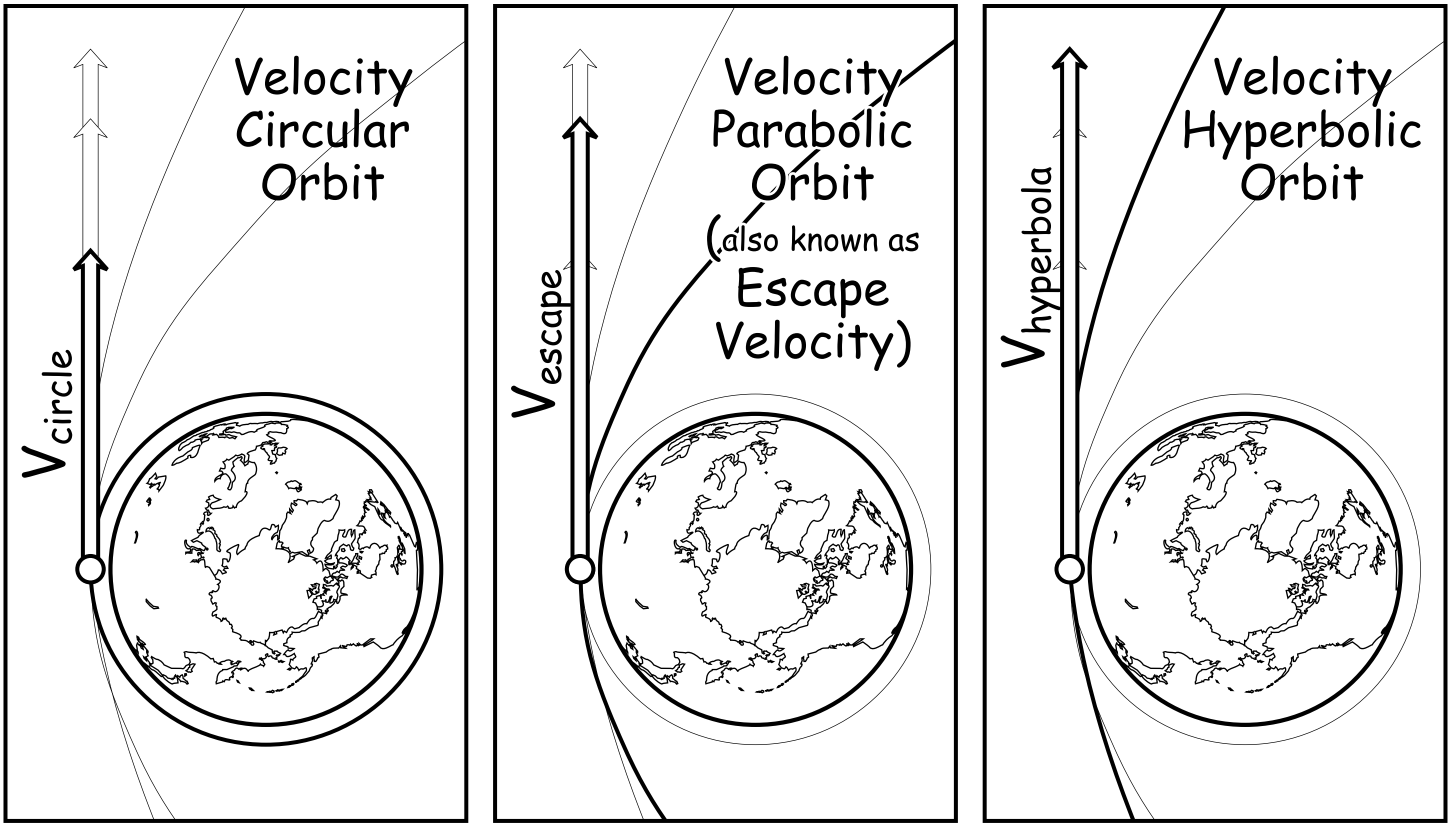
As an object falls towards Earth, it moves faster and faster. At the closest point to the Earth, the perigee, it's moving at top speed. As it moves away, Earth's gravity pulls it, slowing it down. As the hyperbola gets closer to the asymptote, the speed gets closer and closer to **V infinity**, the speed the object would have at an infinite distance from Earth.



After a few million kilometers from the Earth, it is moving so close to V infinity, the difference is negligible.

V infinity is also called the hyperbolic excess speed.


A ship leaves low Earth orbit entering a hyperbolic trajectory with regard to the Earth.
Zooming out we can see that the hyperbolic path approaches a straight line.
We zoom out some more.
What is this? The so called "straight lines" are starting to gently curve.
What's going on here?
We're entering a scale where the tiny Earth's influence is barely visible, **but we can start to see the effects of the much larger sun.**
We zoom out even more and switch reference frames. No longer is the Earth our center. The hyperbola with regard to Earth becomes a tiny germ sitting on a much larger curve: **The sun centered Earth to Mars Hohmann ellipse we saw on page 13.**
The 3 kilometers/sec difference between the ellipse's perihelion velocity around the sun and Earth's circular orbit velocity around the Sun **is the Vinfinity for the hyperbola with regard to Earth.**
3 km/sec
If we zoom in with a microscope and switch reference frames to Mars centered, we'd see another tiny hyperbola with regard to Mars
SUN



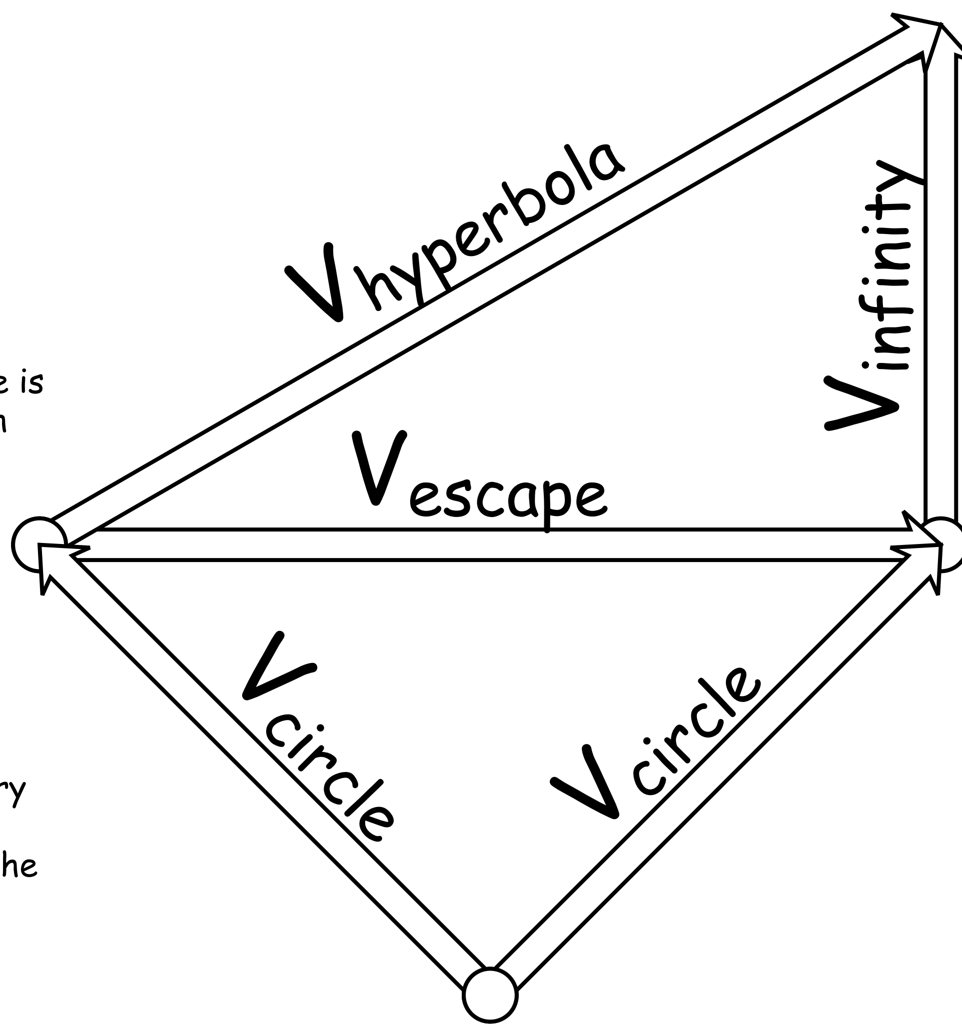
The further from a planet, the slower a circular orbit

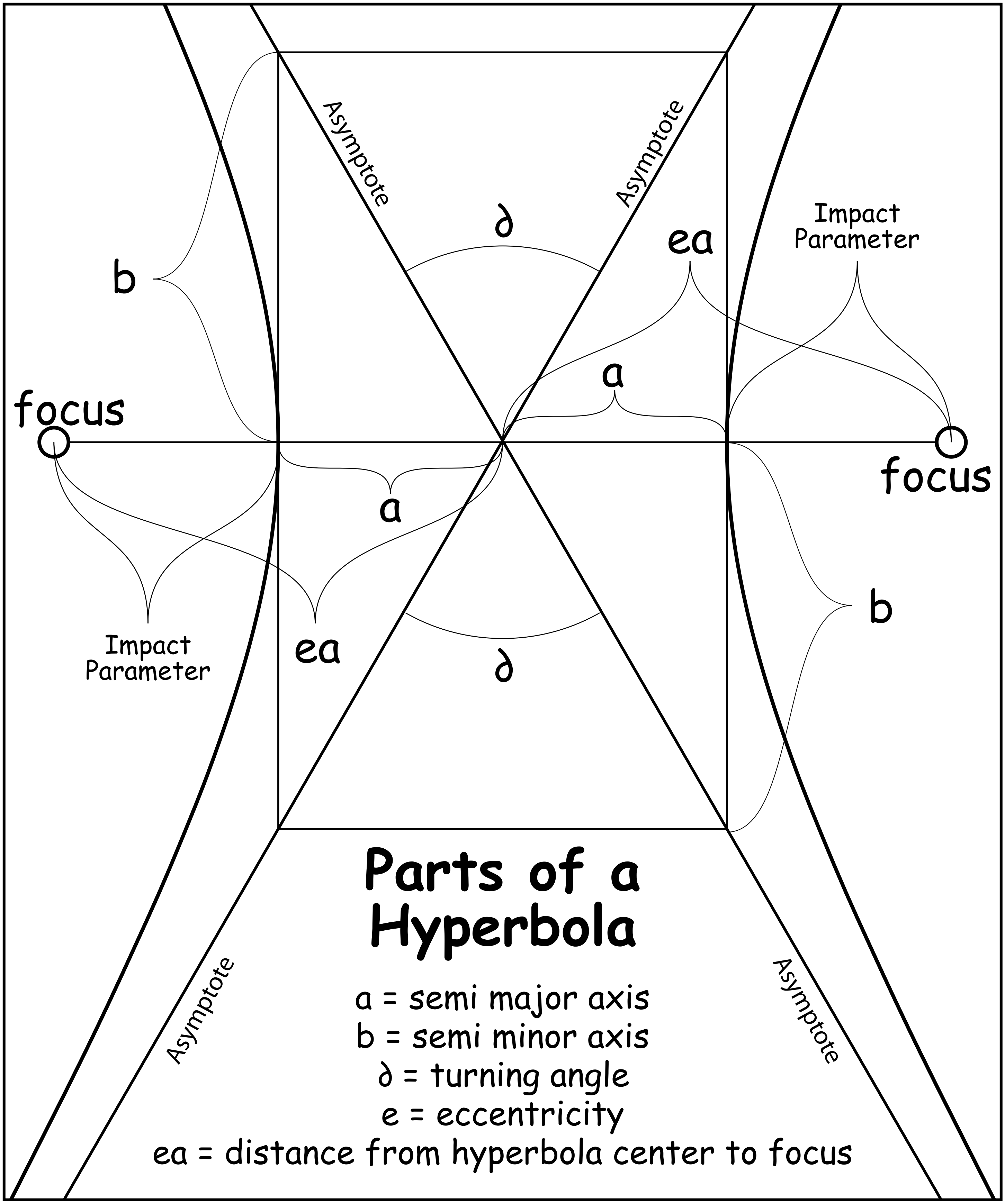
At a given altitude, $V_{esc} = 2^{1/2} \times V_{circ}$.
The square root of 2 is about 1.414.
For a right isosceles triangle, the hypotenuse is $2^{1/2}$ x the length of each of the two equal legs.

Likewise, $V_{hyp}^2 = V_{esc}^2 + V_{inf}^2$.

Remember the **Pythagorean Theorem** on pages 22 and 23?

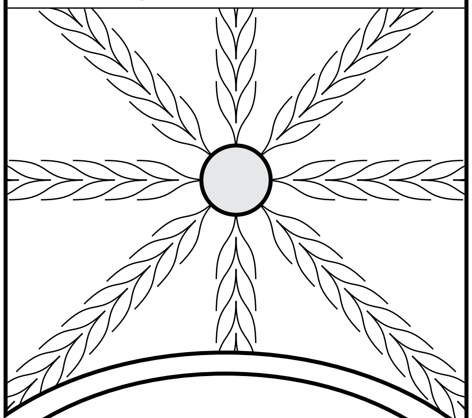
Using the Pythagorean Theorem and the memory device to the right, it's not hard to remember the relationships between V_{circ}, V_{esc}, V_{hyp}, and V_{inf}.





The semi major axis of a hyperbola is often denoted with the letter a. This is a negative number. A hyperbola's eccentricity is often labeled e.

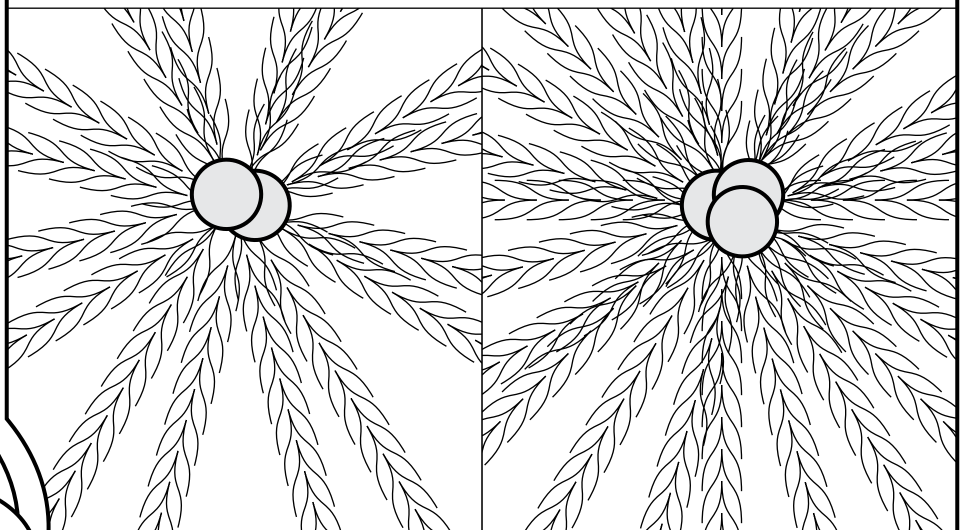
Each speck of matter pulls other specks. You can think of gravity as each speck sending out tractor beams

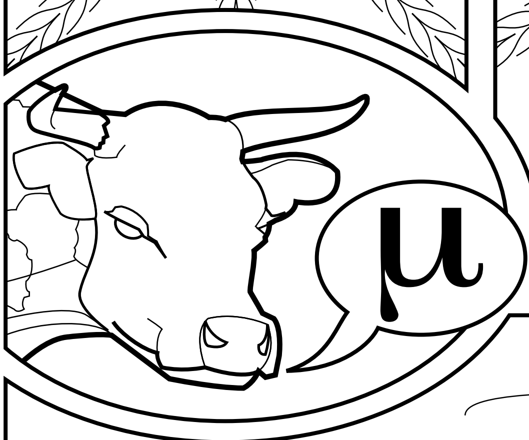


More specks, more "tractor beams". The more mass, the stronger the pull.

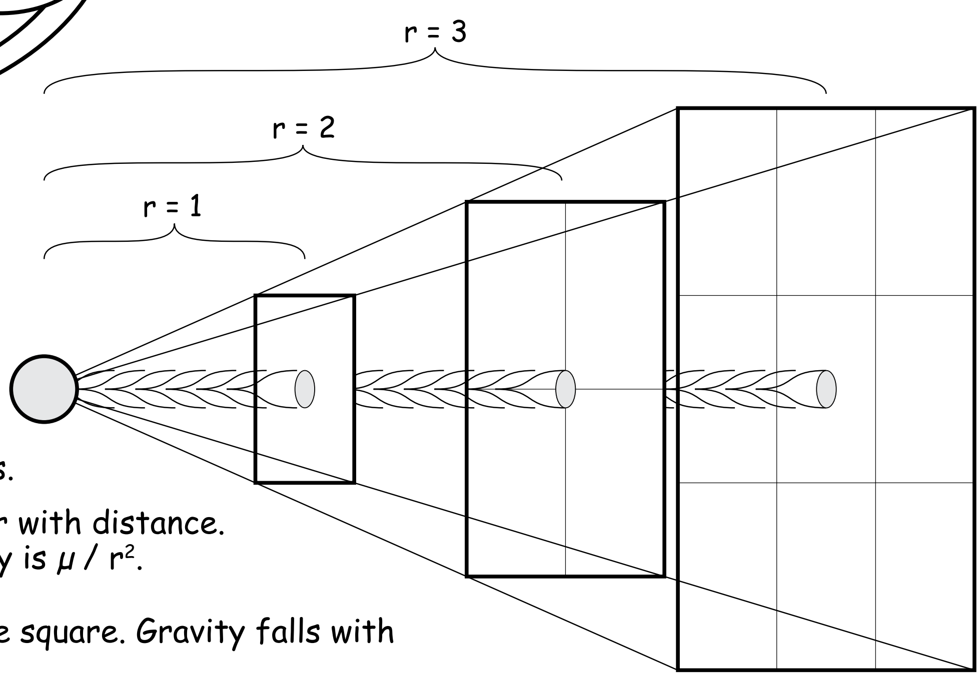
A body's pull is G x mass.

G is always the same, Mass is the amount of matter in a body.





G x mass is often called μ, pronounced "Moo"



At distance r = 1, there's 1 tractor beam per square unit.

Double the distance, there's 1 beam per 4 square units

Triple the distance, 1 beam per 9 square units.

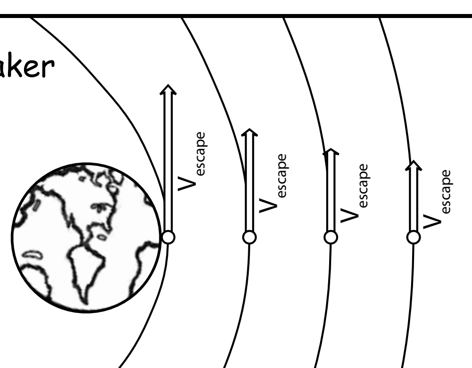
Gravity's pull gets weaker with distance. Acceleration from gravity is μ / r^2.

$1 / r^2$ is called the inverse square. Gravity falls with the inverse square or r.

Since gravity gets weaker with more distance, it takes less speed to escape.

Escape velocity is $(2 \times \mu / r)^{1/2}$.

V infinity is $(\mu / -a)^{1/2}$.



From page 46: $V_{hyp}{}^2 = V_{esc}{}^2 + V_{inf}{}^2$.

Substituting for V_{esc} and V_{inf}:

$V_{hyp}{}^2 = ((2 \times \mu / r)^{1/2})^2 + ((\mu / -a)^{1/2})^2$.

$V_{hyp}{}^2 = (2 \times \mu / r) + (\mu / -a)$.

$V_{hyp}{}^2 = \mu\,(2/r - 1/a)$.

$V_{hyp} = (\mu\,(2/r - 1/a))^{1/2}$.

So if we know r and a, we can find the speed.

This is called the **vis-viva equation**.

$V = (\mu\,(2/r - 1/a))^{1/2}$ also works for ellipses.

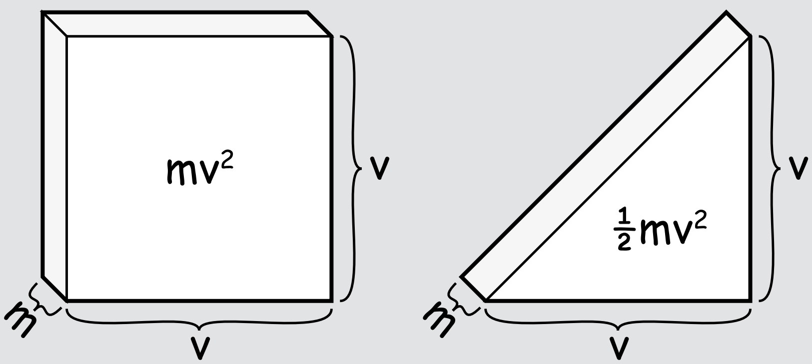
Objects closer to the gravitating body move faster while objects farther away move slower.

The coin funnels you sometimes see at shopping malls can give a feel for orbits. The coin rolls slowly as it starts its path at the edge and coins closer to the center move fast.

Orbiting objects closer don't spiral in, though. Unless it's close enough to earth to feel drag from the earth's atmosphere.

Coin Orbiter image used permission onlinevending.com

Kinetic energy = $\frac{1}{2}mv^2$



Kinetic energy goes with the square of velocity.

Double your speed and you'll quadruple your kinetic energy.

KE also goes with mass. m = mass of the moving object.

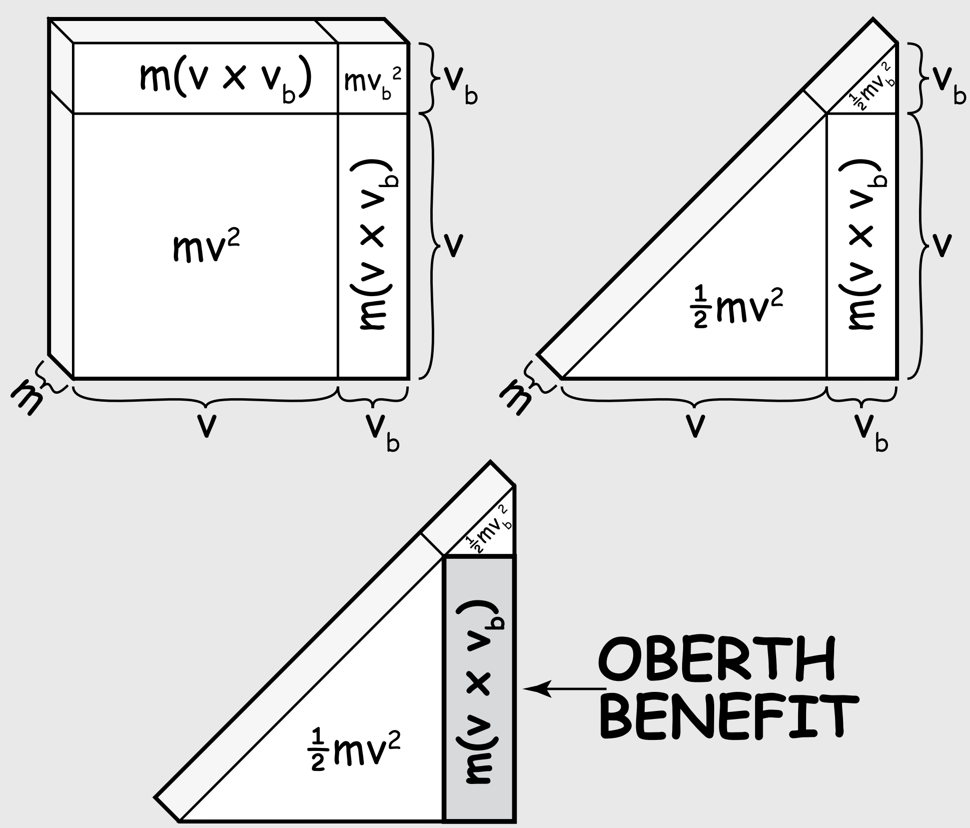
V_b=velocity added by a rocket burn.

If you make a burn to accelerate a rocket while going fast, you get more kinetic energy.

This is known as the

Oberth benefit.

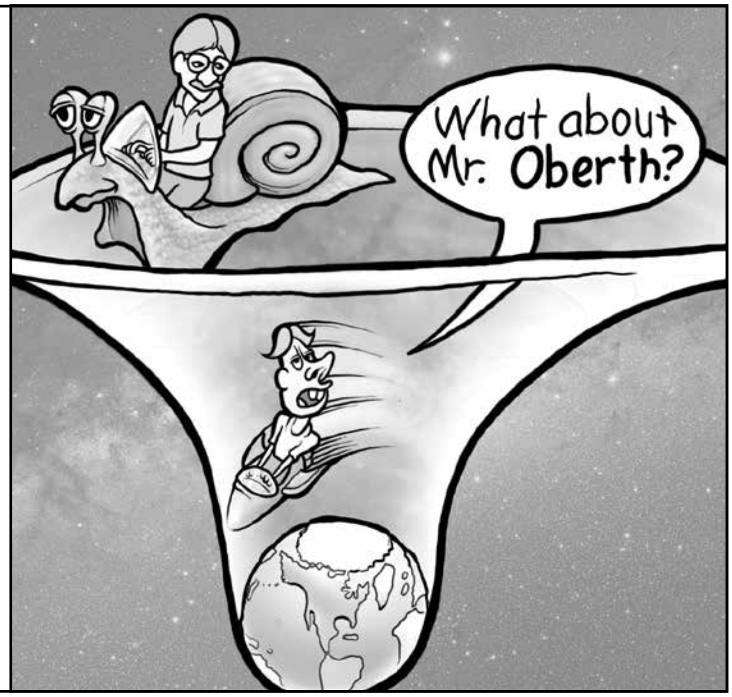
Thus you get more bang for your buck doing a burn when you're closer to a planet and moving faster.

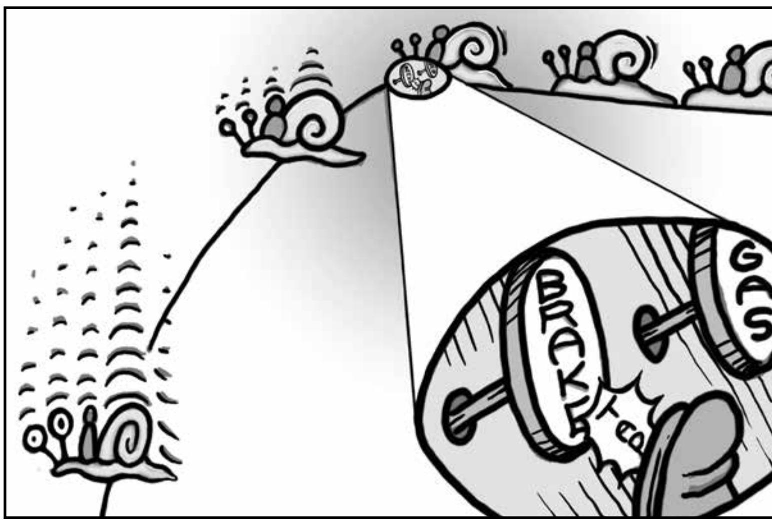


High earth orbits
are relatively slow and
low earth orbits move faster.

So a fellow who calls himself Rune
was telling me it's better
to depart from
LEO (Low Earth Orbit)
when heading for Mars.

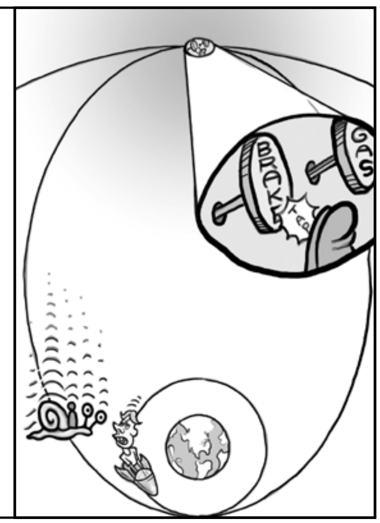
"What about Mr. Oberth?"
Rune asked me.



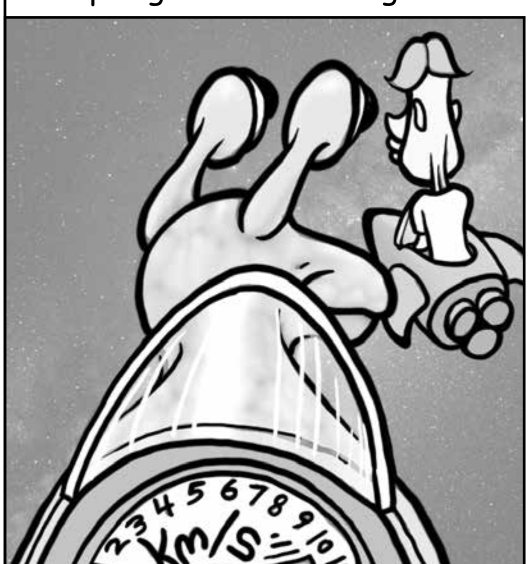


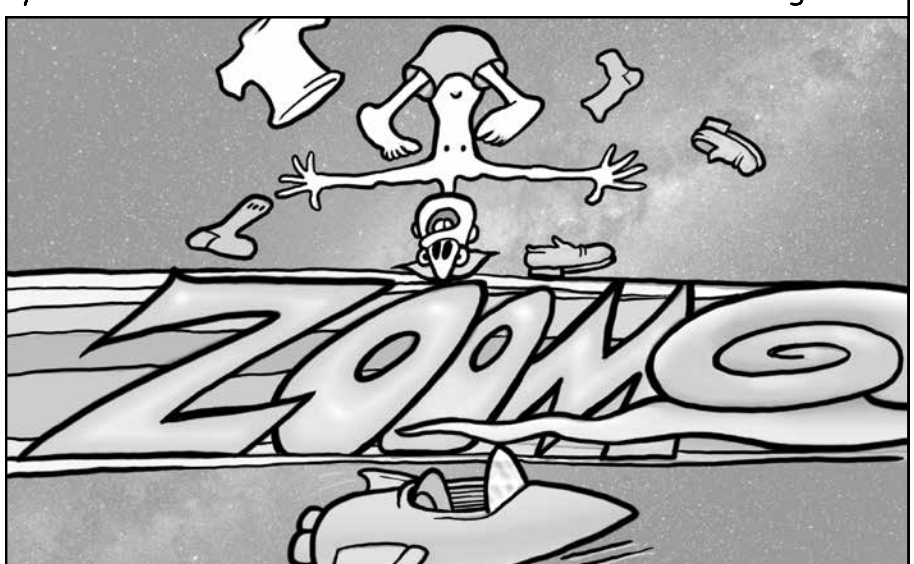
I'm going so slow
that a small tap of
my brakes kills
most my speed
and I start falling
towards earth.

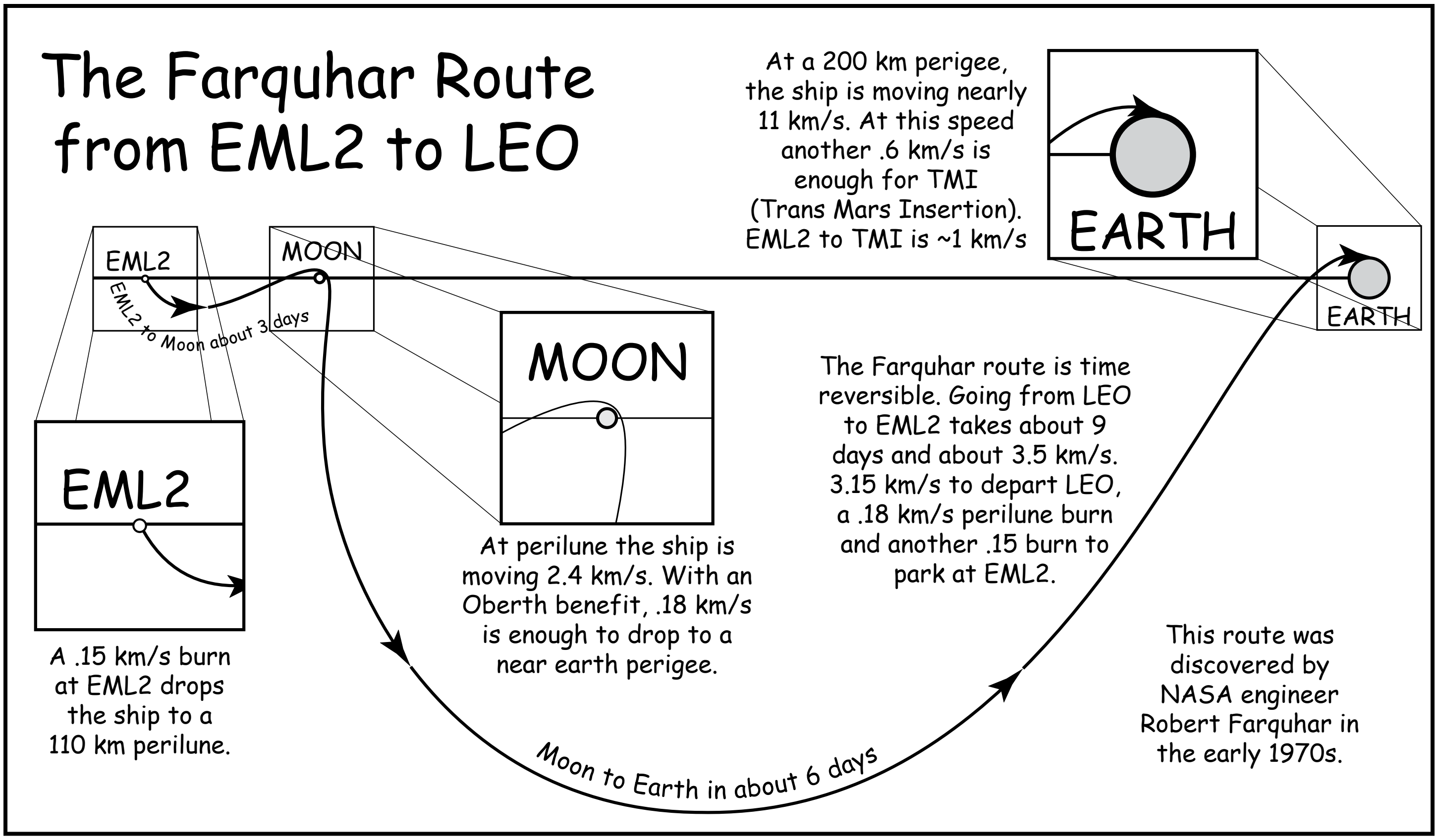
I pick up speed as
I fall towards
perigee (the
closest point to
earth in my
new orbit).

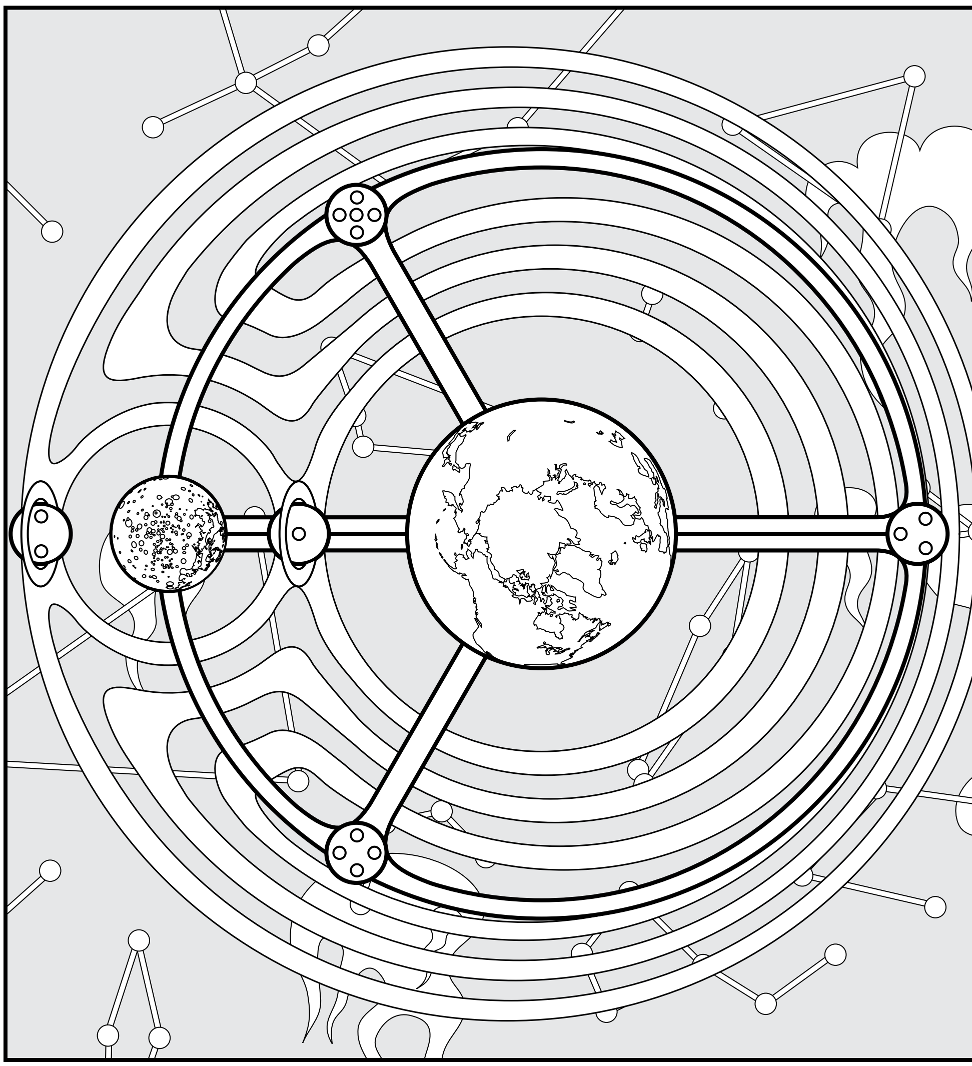


I catch up to Rune at just a hair under escape velocity - 10.9 km/s. Rune is moving 7.7 km/s.
A perigee burn would get me nearly twice the Oberth benefit Rune's LEO burn would give.





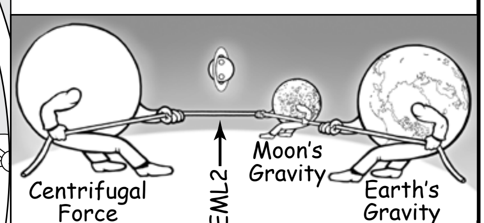




"What's EML2?" you might ask.

EML2 is the 2nd Earth Moon Lagrange Point. There are 5 such points.

These are where the **moon's gravity**, **earth's gravity** and **centrifugal** force all cancel out.



For the EML2 tug-of-war, Earth's gravity & Moon's gravity are on the same team against centrifugal force

Stuff parked at EML4 & 5 tend to stay put.

EML1, 2 & 3 are quasi stable. Stuff parked there will stick around with a small station keeping expense.

In terms of orbital energy, EML2 is the closest to escape.

The Rocket Equation:

Mass fraction propellent = $1 - e^{-\text{delta V/exhaust velocity}}$.

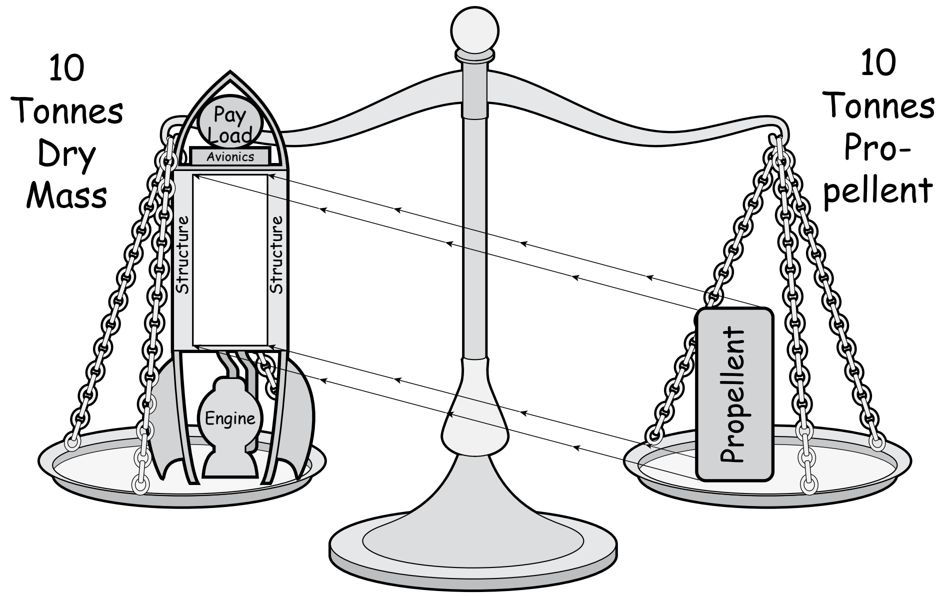
Here the letter e doesn't refer to eccentricity but rather **Euler's number**, a number discovered by Leonhard Euler. The number e is about 2.72

Let's say our **delta V budget is 3 km/s** and we're using oxygen/hydrogen bipropellent with an **exhaust velocity of 4.4 km/s**.

$e^{-(3\text{ km/s}) / (4.4\text{ km/s})} = e^{-3/4.4}$

$= .5057$ (about 1/2)

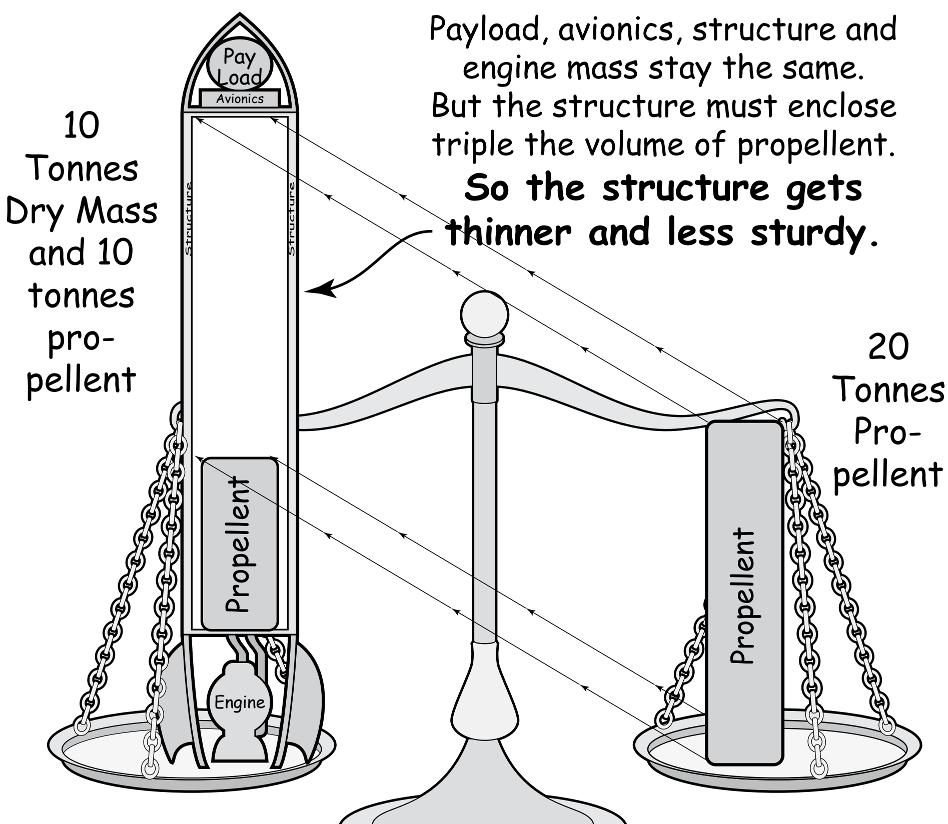
A 3 km/s rocket is about 1/2 propellent by mass.



So if we want a **6 km/s delta V budget,** we need to accelerate 3 km/s more. We need **20 tonnes propellent** to accelerate our 10 tonnes of dry mass plus 10 tonnes of propellent.

Each 3 km/s added to the delta V budget doubles total mass.

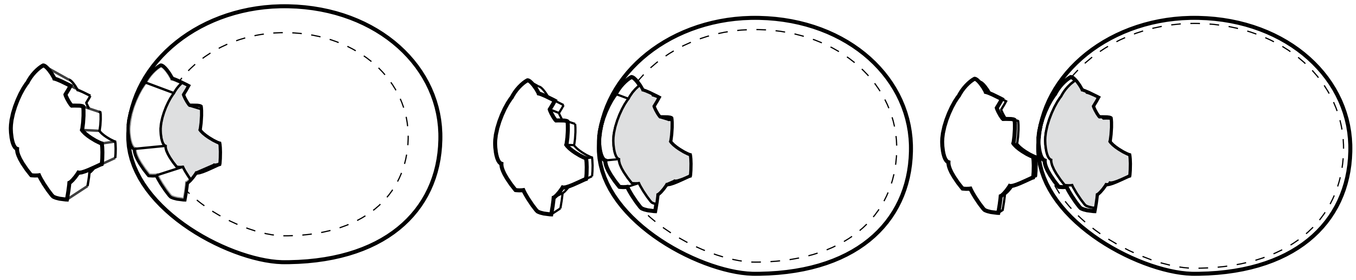
Payload, avionics, structure and engine mass stay the same. But the structure must enclose triple the volume of propellent. **So the structure gets thinner and less sturdy.**



3 km/s
Dry Mass 50%

6 km/s
Dry Mass 25%

9 km/s
Dry Mass 12.5%



One half of this egg's volume is shell.

One fourth of this egg's volume is shell.

One eighth of this egg's volume is shell.

As the delta V budget goes up, the structure of the ship must become thinner and more delicate. It takes between 9 and 10 km/s to get to orbit and between 12 and 13 km/s to earth escape. So the upper stages must have walls and structure egg shell thin.

And spacecraft must endure extreme conditions.
Max Q for ascent through earth's atmosphere is often around 35 kilopascals.
For re-entry Max Q can reach 90 kilopascals.

A severe hurricane is about 3 kilopascals.

To meet mass fraction constraints, aerospace engineers have designed staged rockets. Dry mass is thrown away enroute.

Could you imagine how much a transcontinental flight would cost if we threw away a 747 each trip?

The cartoon to the right is somewhat dated. As of this writing (2019) Jeff Bezos' Blue Origin and Elon Musk's SpaceX seem well on their way to making economical, reusable boosters.

But upper stages remain expendable (in other words, disposable).

In a world with no gas stations...

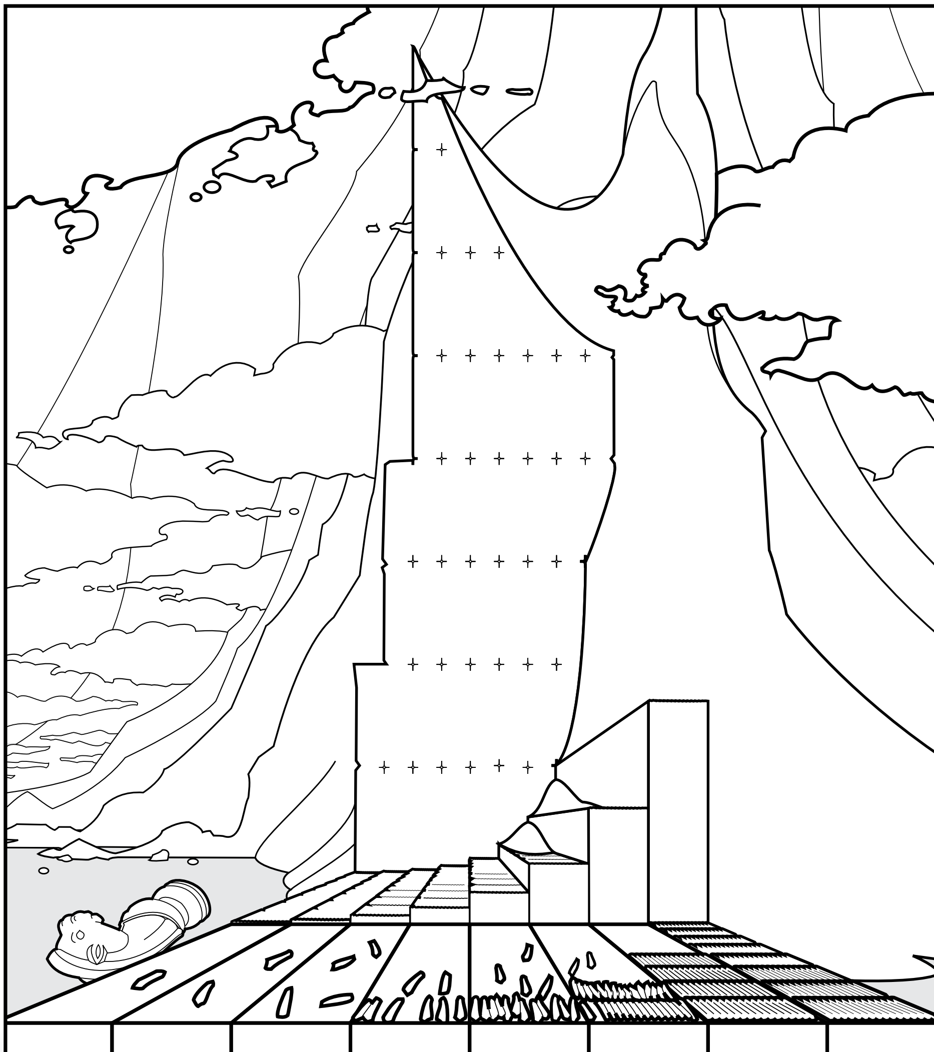
After the tanker fuel is used up, the tank and large engine is dead weight that uses up too much fuel.

It's thrown away

After the pickup does it's part, it's tossed.

The VW bug meets the same fate...

And the motorcycle gets flushed. For decades this has been the way to reach destinations.



The exponential rocket equation

$Mass_{Propellent} = Mass_{Final}\ e^{(Dv/Vexhaust)} - 1$

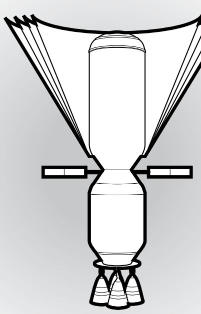
The Legend of Pal Paysam illustrates the power of exponential growth.

An east Indian king was proud of his chess playing skills. A stranger (Krishna in disguise) challenged the king to a game with this wager: 1 rice grain on the first square of the chess board, 2 grains on the second square, then 4 and continuing to double each square of the board.

Only after the king lost did he realize the enormity of his wager.

$2^{65} - 1 =$
36893488147410103231
grains of rice

Which would be about 7 times the mass of Mount Everest.



Propellent Depots

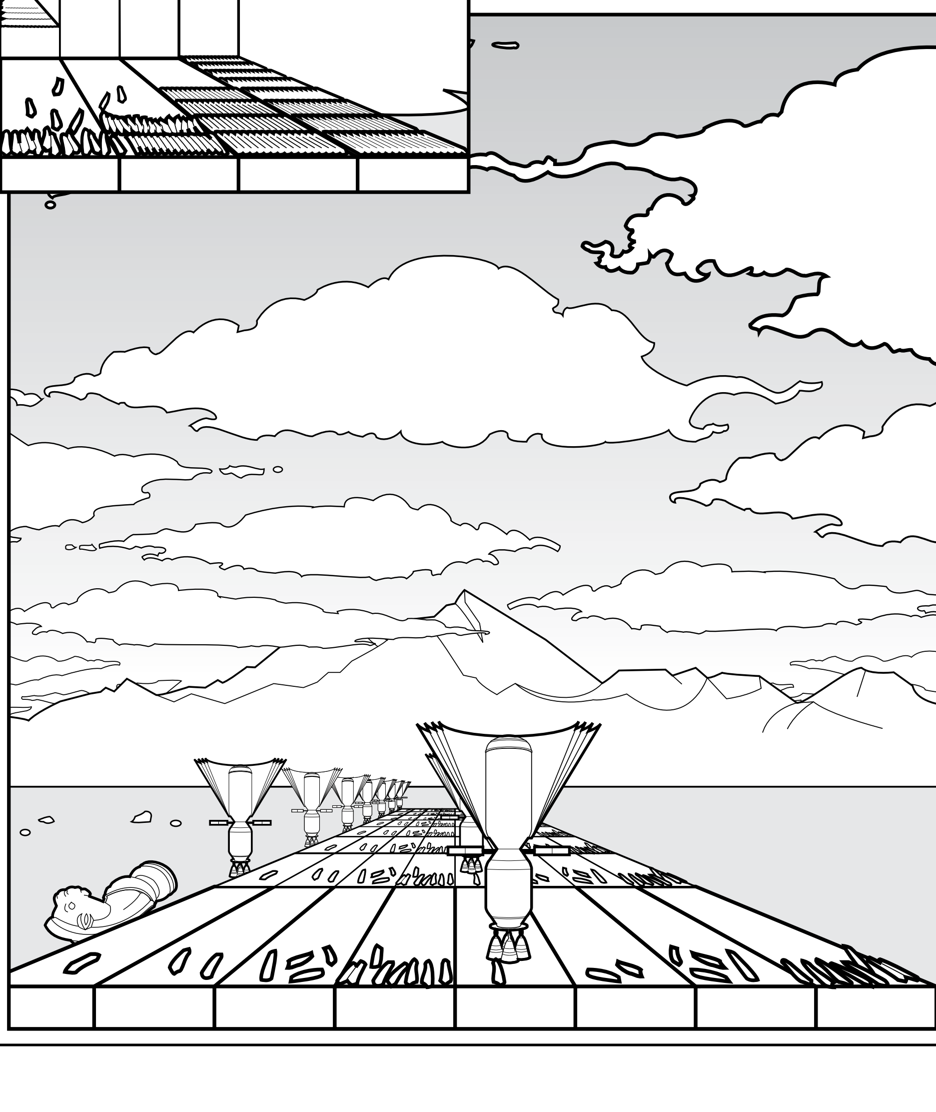
Given a propellent depot every so often and the exponent in the rocket equation is broken into smaller chunks.

$2^2 + 2^2 + 2^2 + 2^2 << 2^8$
For example.

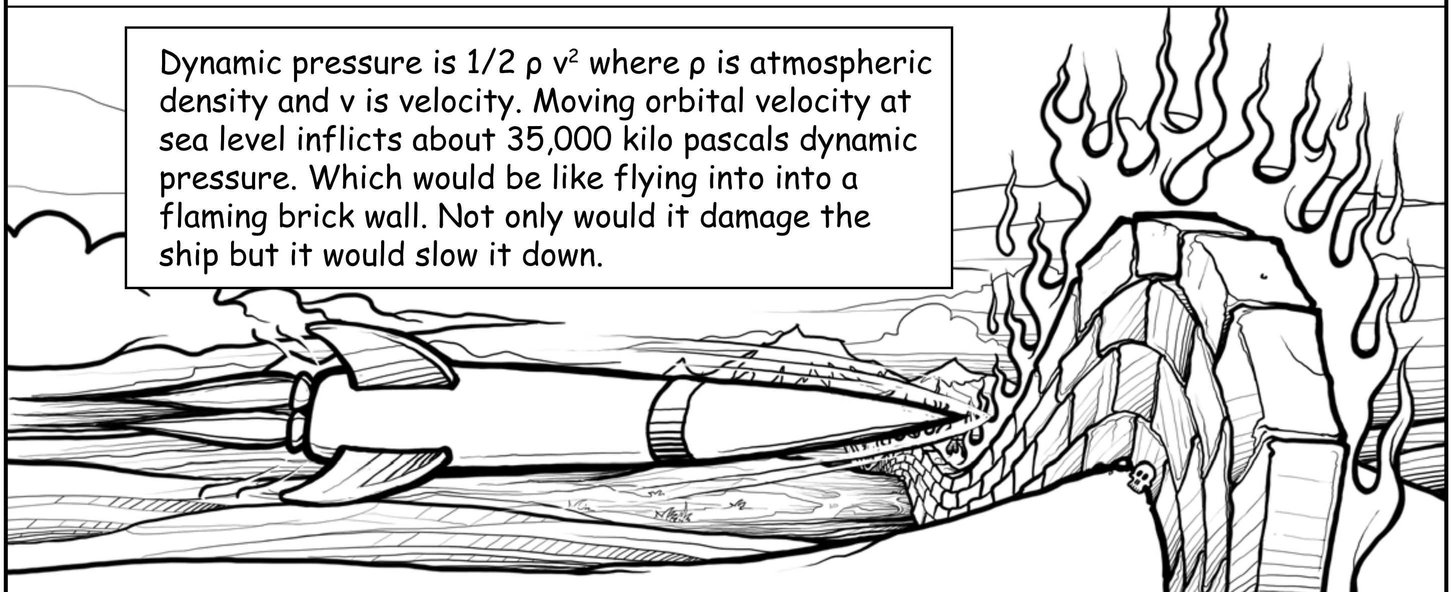
To the right Mt. Everest can now be seen in the background.

Refueling every so often saves a bunch of propellent.

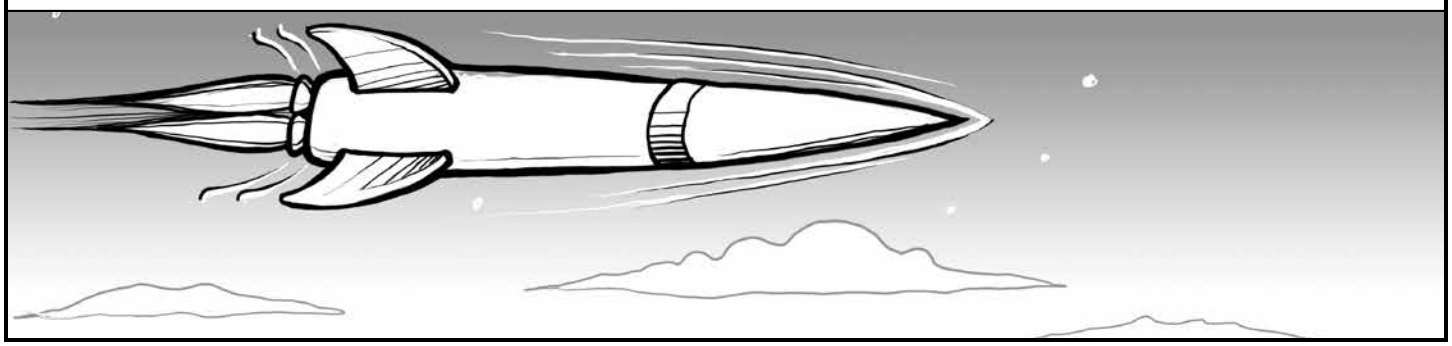
More importantly, breaking the delta V budget into smaller chunks makes for more doable mass fractions.

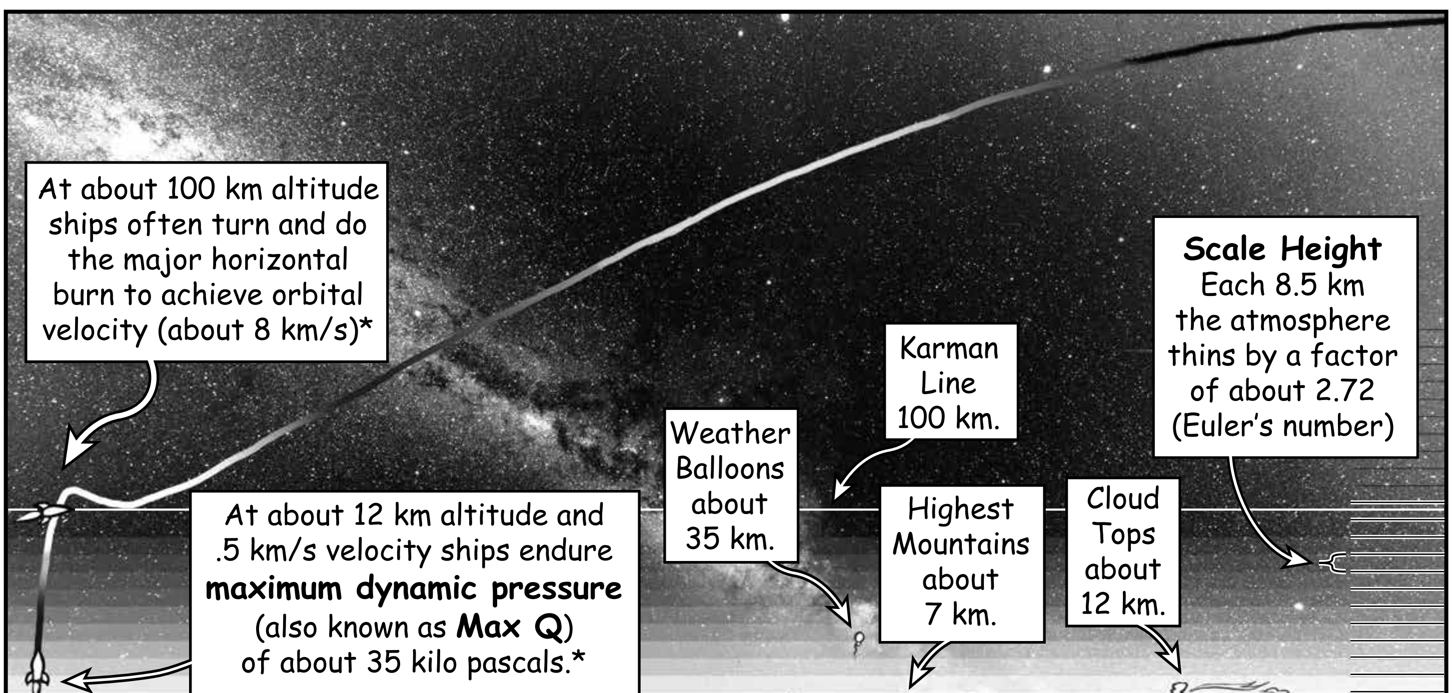


A severe hurricane is about 3 kilo pascals. Typical Max Q for a rocket's ascent is about 35 kilo pascals. Moving orbital velocity at sea level inflicts about 35,000 kilopascals.



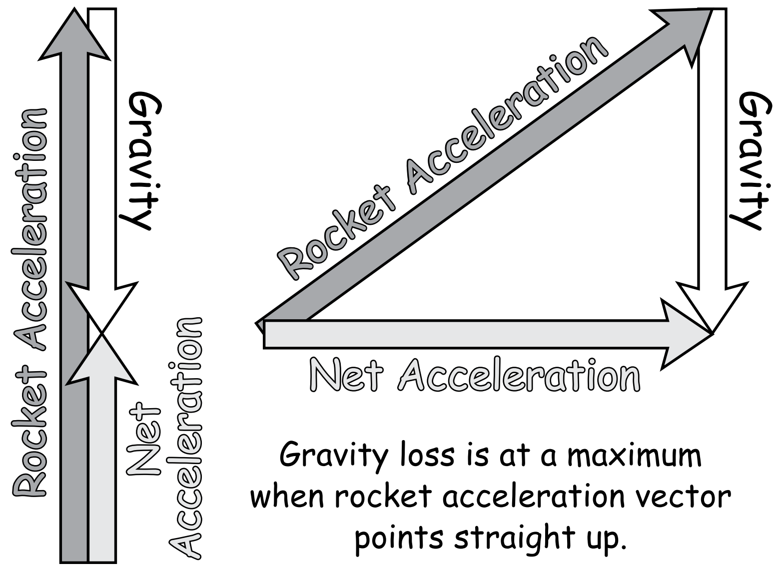
At 100 km altitude the air's so thin the ship suffers little dynamic pressure. Ships usually attain this altitude before doing the major burn to achieve orbital velocity.





*Numbers are approximate. Ships can reach Max Q or do burns at different altitudes & velocities.

GRAVITY LOSS



Gravity loss is at a maximum when rocket acceleration vector points straight up.

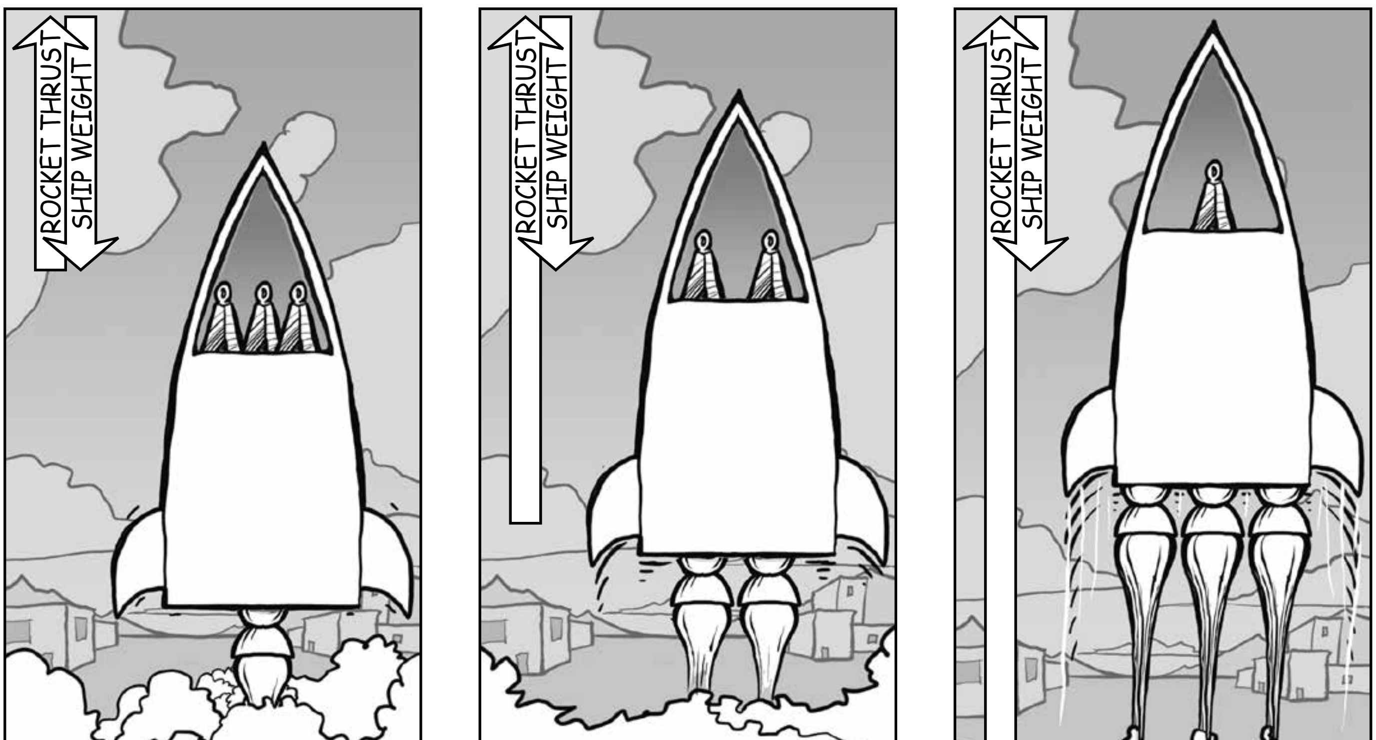
Gravity cancels out some of a rocket's upward acceleration.

Earth surface gravity: 9.8 m/sec^2.

102 seconds vertical ascent means 1 km/s gravity loss. To minimize gravity loss, ascent needs to be as fast as possible.

For ascent we want to maximize thrust & acceleration. A booster stage will typically have more rocket engines than an upper stage.

THRUST/WEIGHT RATIO (T/W)



T/W = 1
The ship hovers in place.
It never gets off the ground.

T/W = 2
It takes the ship 143 seconds
to reach the Karman Line.

T/W = 3
It takes the ship 101 seconds
to reach the Karman Line.

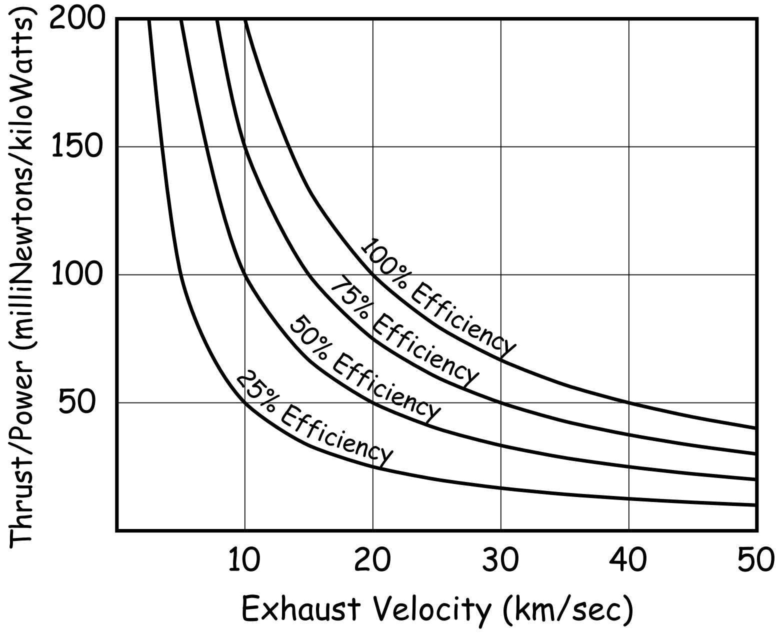
THE MYTH OF 30X — The Tier One Project won the $10 million Ansari X-Prize in 2004 when they made two suborbital trips within 5 days with a reusable manned rocket. Some said "Big deal. Potential energy at the Karman line is only 1/30 of the kinetic energy of a 7.7 km/s orbit. Getting altitude isn't the problem -- It's going sideways fast." This argument ignores gravity loss and a booster's need for extra thrust. A booster stage to get above the Karman line can easily be 2/3 of a rocket's cost.

Thrust vs Exhaust Velocity

A rocket with higher **exhaust velocity** can achieve more delta V with a lower propellent mass fraction. High ISP propellent is desirable.

However high **thrust** is also desirable. We need a high trust to weight ratio to climb above earth's atmosphere without exorbitant gravity loss.

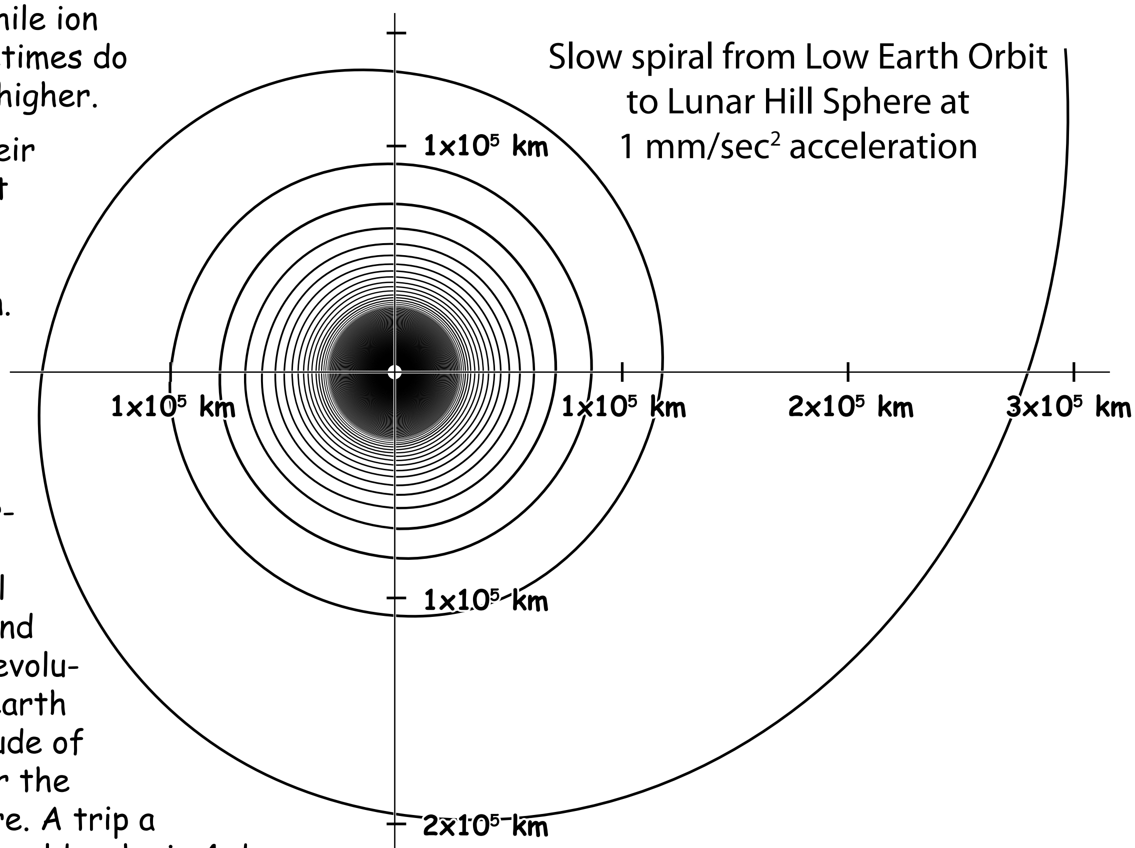
Sadly thrust goes down when exhaust velocity goes up. To the right is a graph showing an ideal ion engine's thrust to power for different exhaust velocities.



Best chemical exhaust velocity is around 4 km/s while ion engines can sometimes do up to 30 km/s & higher.

However with their very low thrust it can take an ion rocket a loooong time to do a burn.

To the right an ion rocket starts at a 400 km altitude circular orbit and accelerates at 1 mm/sec^2. It will take it 75 days and more than 345 revolutions about the earth to reach an altitude of 300,000 km, near the moon's Hill Sphere. A trip a chemical rocket could make in 4 days.



Ion engines can have a much higher exhaust velocity but with the lower acceleration it takes much longer to achieve a change in velocity. Since much of the acceleration is done higher on the slopes of a planetary gravity well, there is less Oberth benefit.

What's a milliNewton?

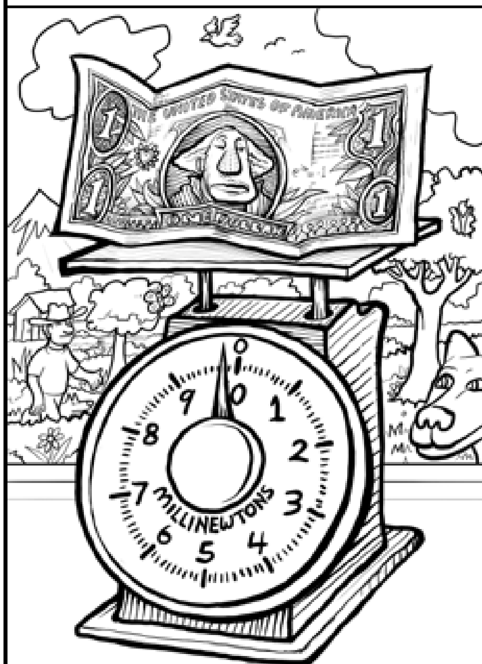
A newton is a unit of force. And force is mass times acceleration.

1 newton = 1 kilogram * 1 meter/second^2.

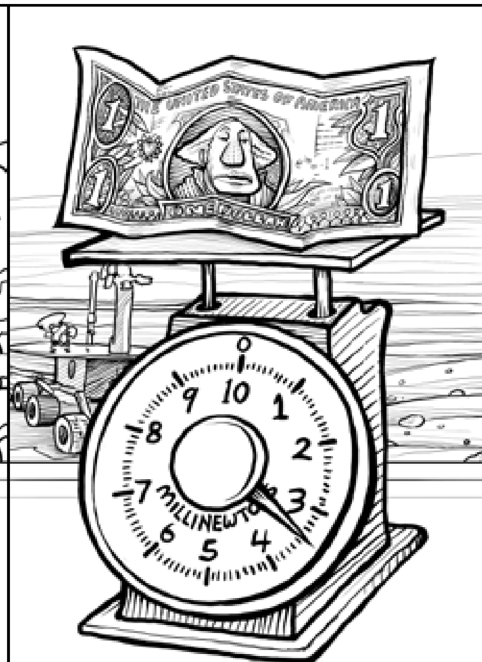
A millinewton is 1/1000 of a newton.

1 millinewton = 1 gram * 1 meter/second^2.

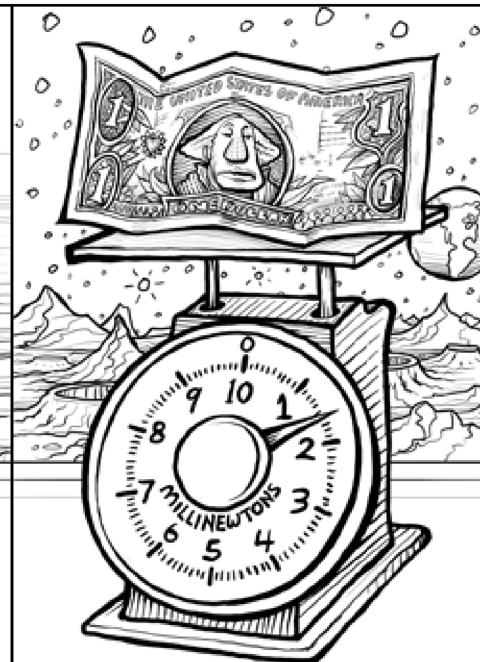
A dollar bill has a mass of one gram.



Earth's surface gravity is 9.8 meters/second^2. On earth a one gram dollar bill weighs 9.8 millinewtons.



Mars' surface gravity is 3.4 meters/second^2. On Mars a one gram dollar bill weighs 3.4 millinewtons.



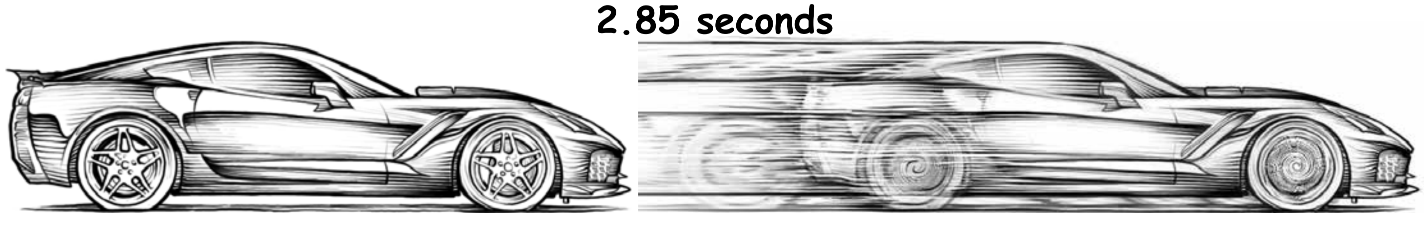
Luna's surface gravity is 1.6 meters/second^2. On the moon a one gram dollar bill weighs 1.6 millinewtons.

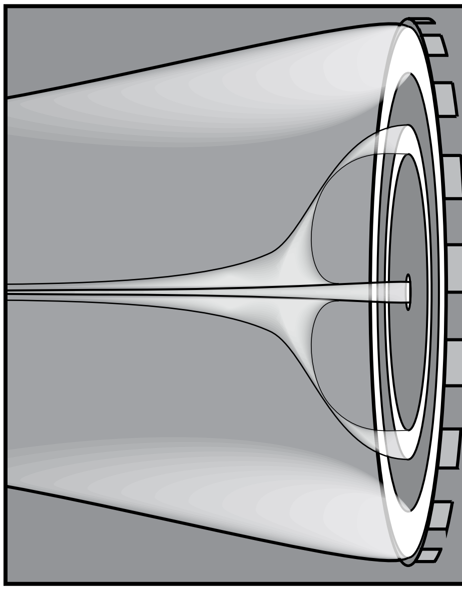
Besides weight, newtons and millinewtons also measure a rocket's thrust.

What's acceleration?

Acceleration is change in velocity over time.
Units can be (meters/second)/second. Which is meters/second^2, or m/s^2 for short.
A 2019 Corvette ZR1 goes from zero to sixty miles per hour in 2.85 seconds
(60 miles/hour) / 2.85 seconds = (60 * 1609 meters/ 3600 seconds) / 2.85 seconds
= **9.4 meters / second^2**. 1 earth gravity is 9.8 m/s^2 so passengers feel
just short of 1 g acceleration when the driver puts the pedal to the metal.

0 to 60 mph in 2.85 seconds





The plume of ionized xenon coming from an XR-100 Hall Thruster is a beautiful thing. The ionized xenon atoms go in different directions at different speeds but the effective exhaust velocity ranges from 16 to 32 km/s.

The XR-100 gives up to 5 newtons of thrust and masses 230 kg. 5 newtons/230 kg is about 21 mm/s^2 acceleration. That seems decent.

But we also need a 100 kilowatt power source.
That can be another 1,400 kg. Add to that structure and avionics, power processing unit and payload and dry mass can total 4000 kg.

Let's say you want an 11 km/s delta V budget. At maximum thrust and 16 km/s exhaust velocity, that's another 4000 kg of xenon.

That's around .6 mm/s^2 for a craft full of xenon and around 1.2 mm/s^2 when xenon's nearly depleted.

A lower mass power source is desirable.

The Need for a Better Alpha

Alpha is a measure of how much mass it takes to generate power.

In 2011 Franklin Chang Diaz caused quite a stir when he claimed his VASIMR ion engine could get men to Mars in 39 days. A typical Hohmann trip to Mars is around 8.5 months.

However Diaz' claims relied on an alpha of .5 kilograms per kilowatt electricity. Kirk Sorensen, Robert Zubrin and others have said such a high power, low mass power source isn't doable.

What is a .5 kg/kWe alpha?.

I try to portray it to the right.

A Ford Focus is 160 horsepower which is 120 kilowatts.

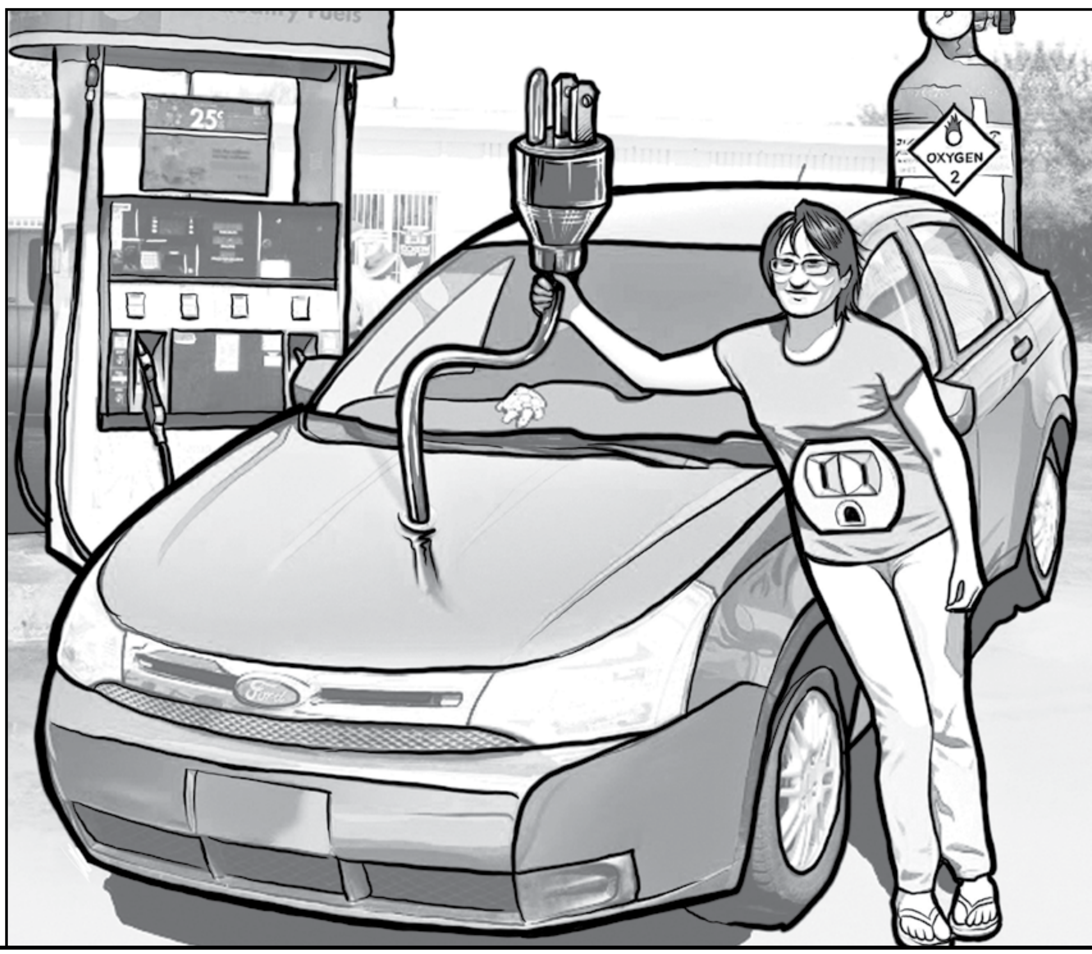
Dominique is 60 kilograms.

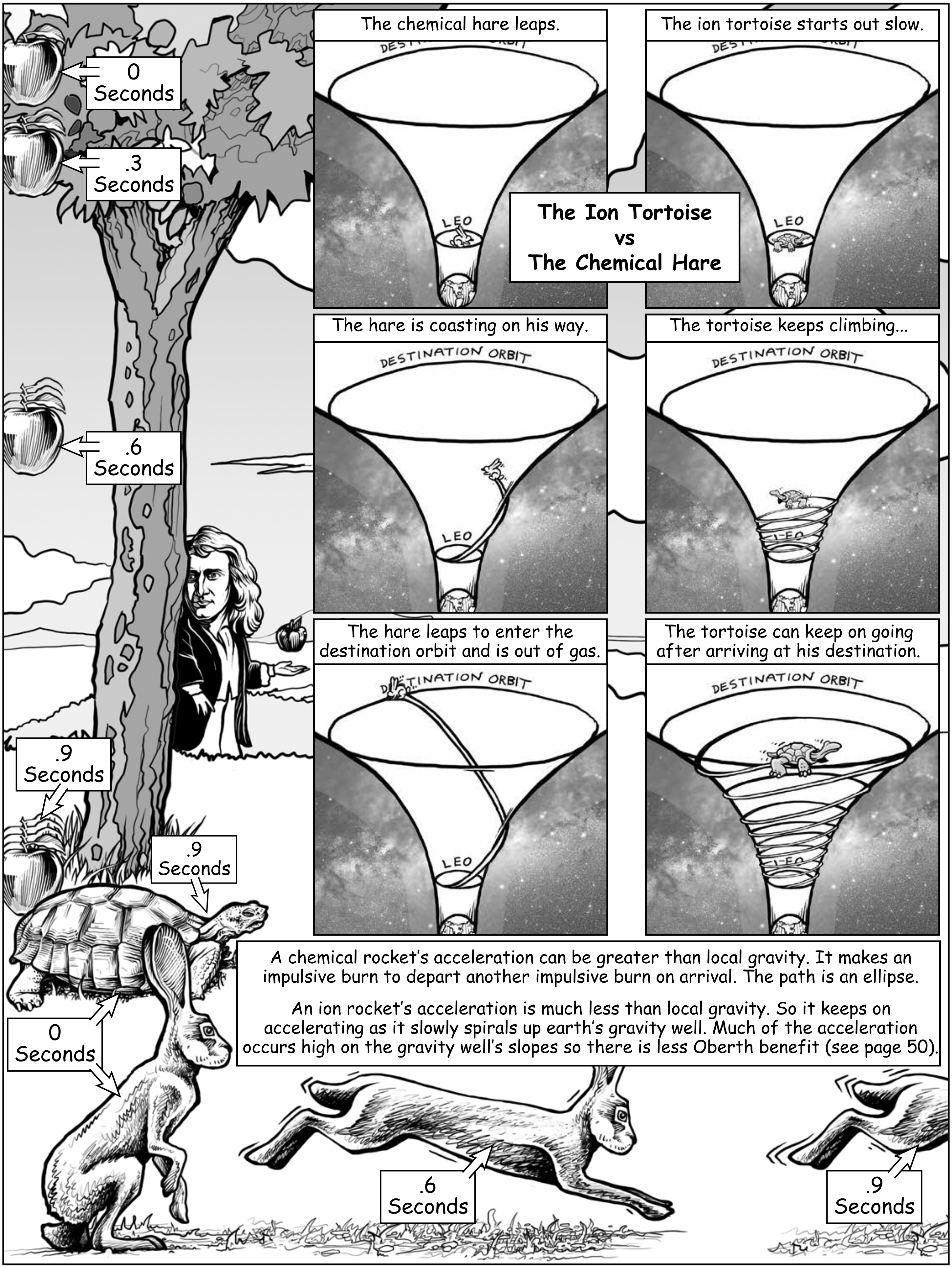
That's .5 kg/kW.

Dominique must also do the work of the gasoline and oxygen the engine burns.

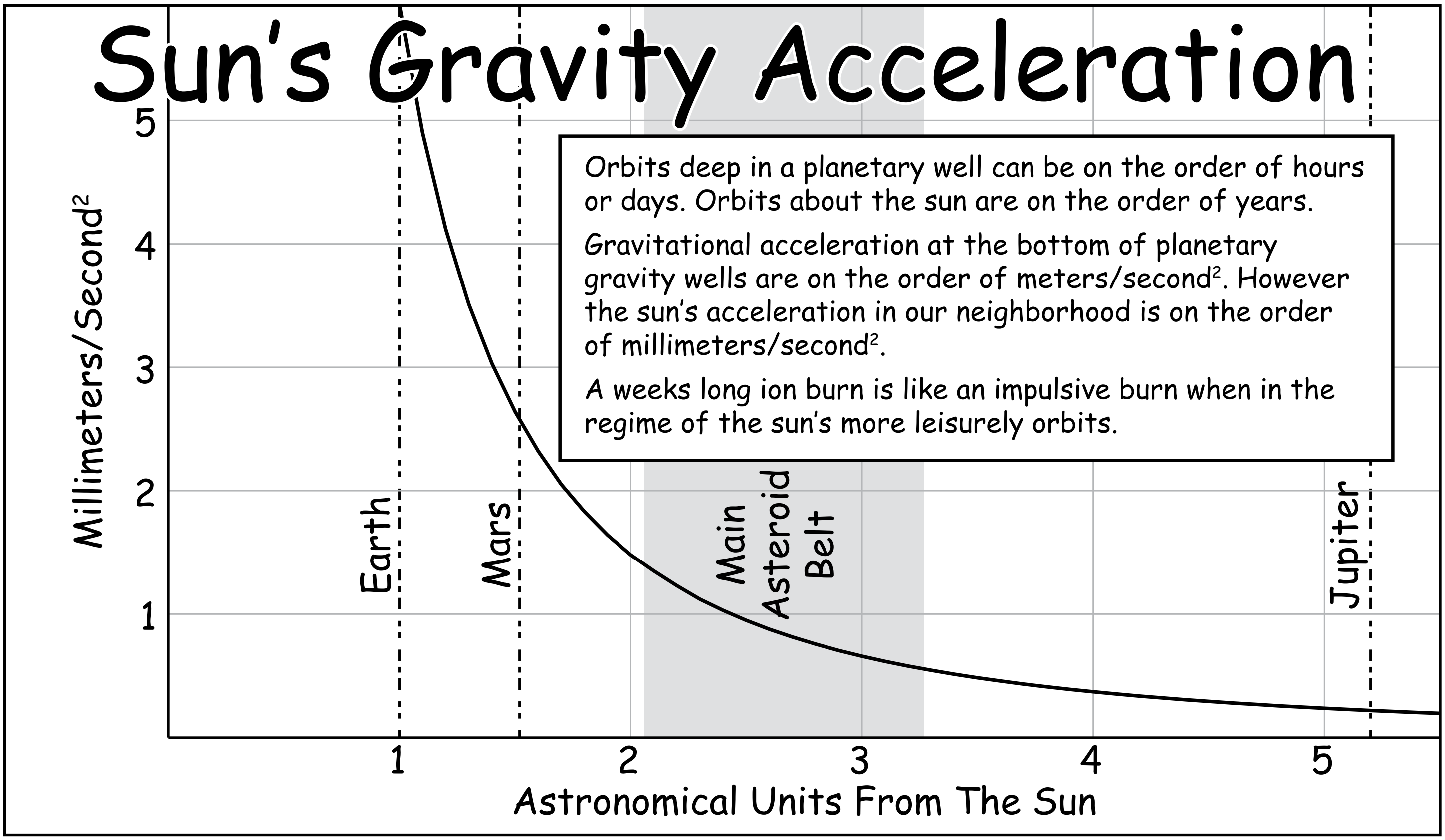
There are no gas stations or charging stations on the way to Mars. Nor is there an oxygen atmosphere.

Is such a power source impossible? I hope not. It's certainly something to strive for.




0 Seconds
.3 Seconds
.6 Seconds
.9 Seconds
.9 Seconds
0 Seconds
.6 Seconds
.9 Seconds
The Ion Tortoise vs The Chemical Hare
The chemical hare leaps.
DESTINATION ORBIT
LEO
The ion tortoise starts out slow.
LEO
The hare is coasting on his way.
DESTINATION ORBIT
LEO
The tortoise keeps climbing...
DESTINATION ORBIT
LEO
The hare leaps to enter the destination orbit and is out of gas.
LEO
The tortoise can keep on going after arriving at his destination.
DESTINATION ORBIT
LEO
A chemical rocket's acceleration can be greater than local gravity. It makes an impulsive burn to depart another impulsive burn on arrival. The path is an ellipse.
An ion rocket's acceleration is much less than local gravity. So it keeps on accelerating as it slowly spirals up earth's gravity well. Much of the acceleration occurs high on the gravity well's slopes so there is less Oberth benefit (see page 50).

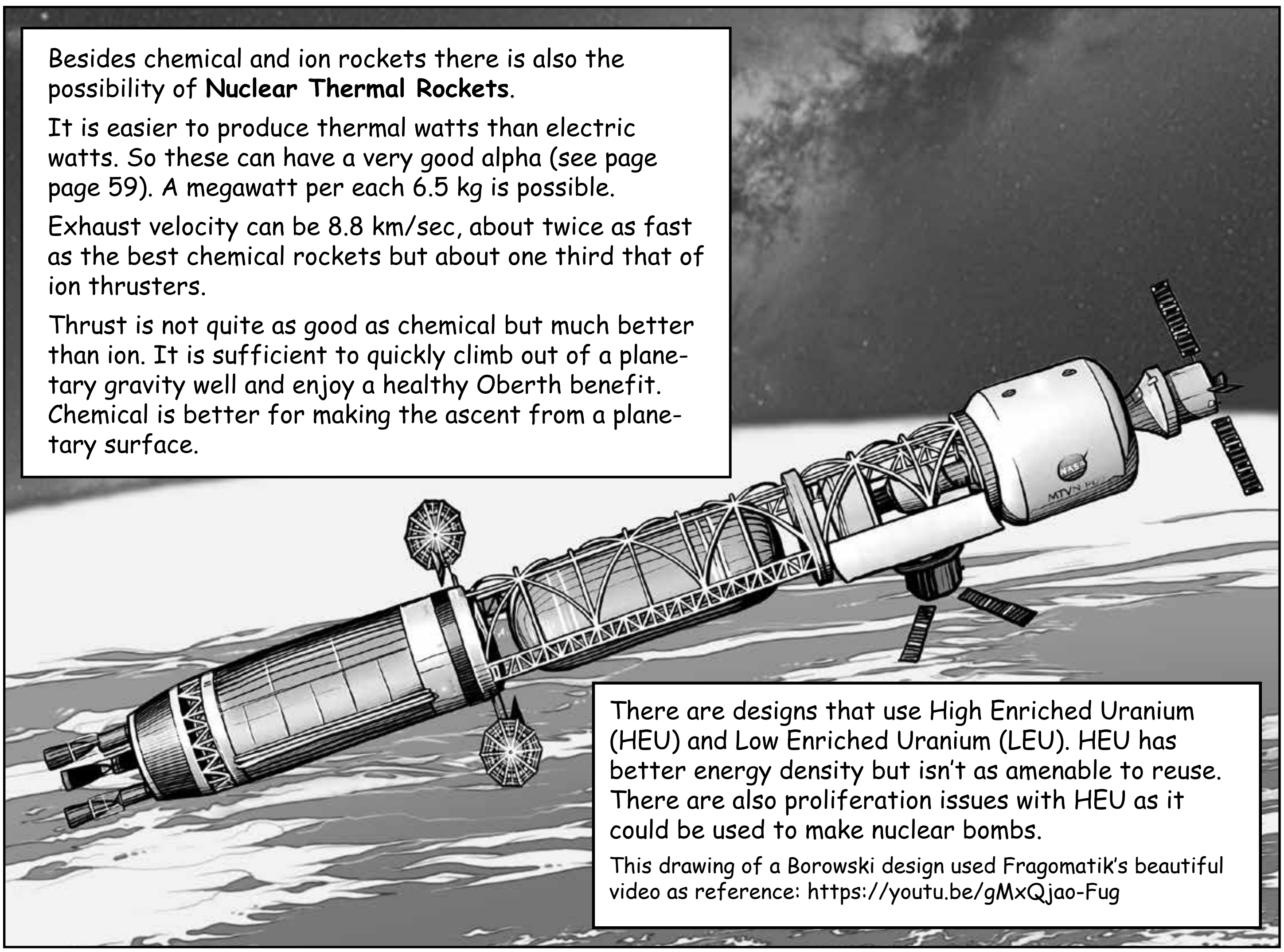
Sun's Gravity Acceleration



Orbits deep in a planetary well can be on the order of hours or days. Orbits about the sun are on the order of years.

Gravitational acceleration at the bottom of planetary gravity wells are on the order of meters/second^2. However the sun's acceleration in our neighborhood is on the order of millimeters/second^2.

A weeks long ion burn is like an impulsive burn when in the regime of the sun's more leisurely orbits.



Besides chemical and ion rockets there is also the possibility of **Nuclear Thermal Rockets**.

It is easier to produce thermal watts than electric watts. So these can have a very good alpha (see page page 59). A megawatt per each 6.5 kg is possible.

Exhaust velocity can be 8.8 km/sec, about twice as fast as the best chemical rockets but about one third that of ion thrusters.

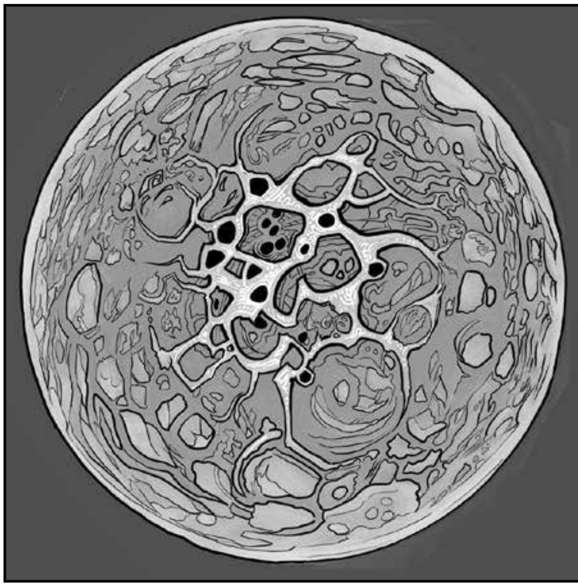
Thrust is not quite as good as chemical but much better than ion. It is sufficient to quickly climb out of a planetary gravity well and enjoy a healthy Oberth benefit. Chemical is better for making the ascent from a planetary surface.

There are designs that use High Enriched Uranium (HEU) and Low Enriched Uranium (LEU). HEU has better energy density but isn't as amenable to reuse. There are also proliferation issues with HEU as it could be used to make nuclear bombs.

This drawing of a Borowski design used Fragomatik's beautiful video as reference: https://youtu.be/gMxQjao-Fug

POTENTIAL PROPELLENT SOURCES

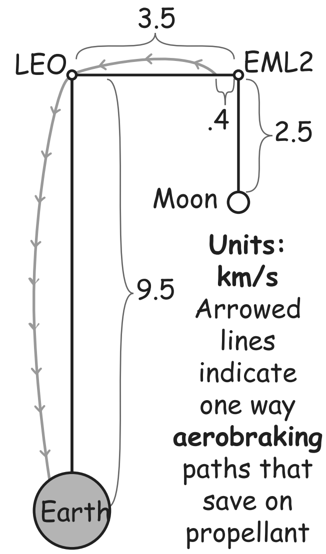
Water ($H_2$0) can be cracked into hydrogen & oxygen, one of the best bipropellants. Carbon dioxide (CO_2) can be cracked into carbon and oxygen. Carbon and hydrogen can make methane (CH_4), one of the more storable rocket fuels. These can be found in various places.



Our Moon's poles have crater floors that never feel sunlight. These can be as cold as 30° Kelvin, cold enough to freeze and trap volatile gases. Probes have detected water and other volatile ices in the cold traps.

Near the cold traps are plateaus that enjoy almost constant sunlight.

A LEO to Moon Hohmann trip is about 5 days. Launch windows to the moon are always open.

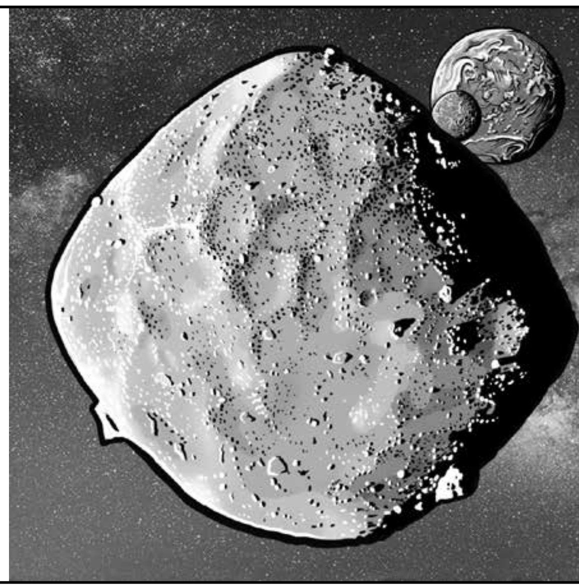


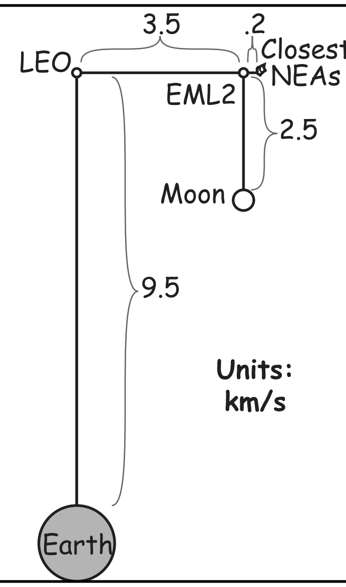
Some Near Earth Asteroids (NEAs) are thought to be 40% water by mass in the form of hydrated clays.

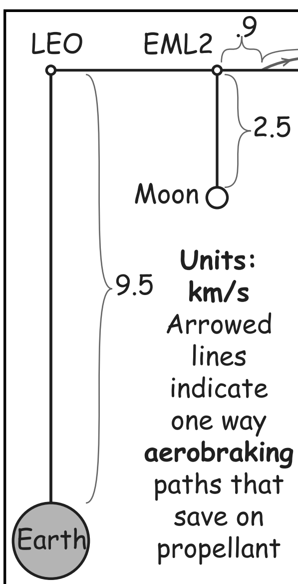
NEAs in heliocentric orbits have rare launch windows and long trip times.

However they can be nudged into loose lunar capture orbits where they would enjoy short trip times and frequent launch windows, just like the moon.

There are NEAs within .2 km/s of EML2.



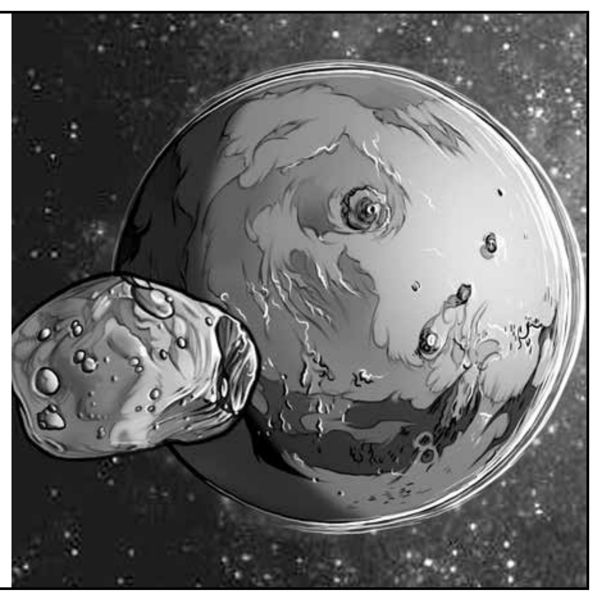


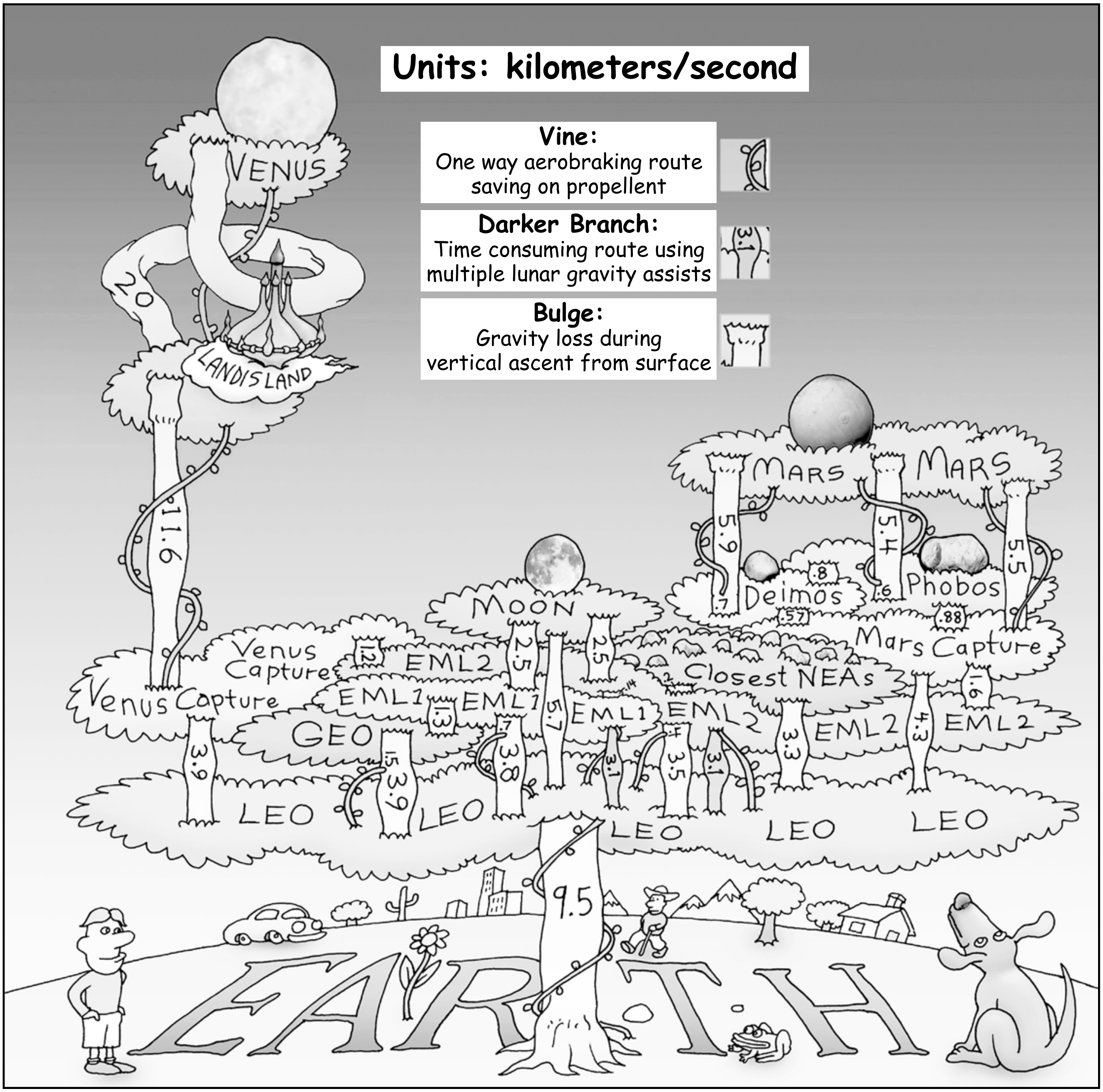


Mars has a thin CO_2 (carbon dioxide) atmosphere, underground H_2O and sheets of H_2O & CO_2 ice at the poles. Mars is about 1 km/s from EML2 (using aerocapture).

Martian moons Phobos & Deimos may also Have propellent. Whether they have water is still unknown.

Hohmann trip time is about 8.5 months and Hohmann launch windows open about every 2.14 years.





Most of these delta Vs are figured using two equations:
The Vis-Viva equation: $V^2 = GM\ (2/r - 1/a)$ and Velocity of Hyperbolic Orbit: $V_{hyp}^2 = V_{esc}^2 + V_{inf}^2$.

The 3.5 km/s number from EML2 to LEO assumes the Farquhar Route (page 51).

The 1.6 km/s from EML2 to Mars Capture assumes using the Farquhar Route (page 51) and then doing the Trans Mars Injection (TMI) burn at LEO when the ship is moving 11 km/s.

The 1.2 km/s number from EML2 to Venus capture also assumes the Farquhar Route and enjoying a healthy Oberth benefit (pages 49 & 50) for the near Earth burn when the ship's moving 11 km/s.

Ascending from Venus' surface through the thick atmosphere would take lots of delta V, hence the 20 km/s from Venus surface to Landis Land. Landis Land is my term for a potentially habitable layer of Venus' atmosphere where pressure and temperature is human friendly. Balloon cities filled with nitrogen and oxygen would be buoyant in Venus' mostly carbon dioxide atmosphere.

The numbers mostly assume Hohmann transfers with impulsive chemical burns. I also assumed circular, coplanar orbits which simplifies calculations but lessens accuracy. The numbers are ball park estimates.

Helpful Websites and Books

Orbital Mechanics: http://www.braeunig.us/space/orbmech.htm
Nice orbital mechanics resource

Encyclopedia Astronautica: http://astronautix.com
Detailed descriptions of various rocket engines including thrust & exhaust velocity, history, more.

Astrogator's Guild: https://see.com/astrogatorsguild/
Professional astrogators Mike and John talk about space exploration

Atomic Rockets: http://www.projectrho.com/public_html/rocket/
Great resource for space enthusiasts and writers of hard science fiction.

Blog on science fiction and space exploration: http://toughsf.blogspot.com
Matter Beam explores various hard science fiction ideas

Blog on space exploration: https://selenianboondocks.com
Jonathan Goff's blog on possible space technologies

http://spaceflighthistory.blogspot.com
Space historian David F. Portree's informative blog

Sarmount's Opening the High Frontier: http://www.high-frontier.org/author/eaglesarmont/
Sarmount suggested vertical skyhooks in the 1990's.

Moonwards, advocates of lunar settlement: https://www.moonwards.com
Kim Holder and friends explore possible benefits lunar development could offer

https://newpapyrusmagazine.blogspot.com
Marcel Williams' thoughts on space exploration and lunar development

A forum on space exploration: https ://forum.nasaspaceflight.com
News and discussion of space exploration

A forum on space exploration: https://www.reddit.com/r/space/
News and discussion of space exploration

Fragomatik's beautiful Youtube channel on plausible hard science fiction scenarios:
https://www.youtube.com/channel/UCOLio0oKOmtWiIlosl_YB1Q

Space Stack Exchange: https://space.stackexchange.com
Questions and answers on space exploration

Orbiter: http://orbit.medphys.ucl.ac.uk
A space flight simulator

Kerbal Space Program: https://www.kerbalspaceprogram.com
A game that teaches orbital mechanics

Scott Manley's YouTube Channel: https://www.youtube.com/user/szyzyg/featured
Kerbal Space Program tutorials and more

Fundamentals of Astrodynamics by Bate, Mueller and White
An inexpensive textbook on orbital mechanics

Nick Stevens space graphics: https://nick-stevens.com/the-artist/professional-work/
Some great illustrations and videos of possible spaceships.

The Worlds of David Darling: http://www.daviddarling.info
Lots of info on music, history, science and math

Mining The Sky by John S. Lewis
Possible resources from the asteroids

Rain of Iron and Ice by John S. Lewis
The possibility of destruction from asteroid impacts